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CONTENTS

ANALYTICAL DESIGN OF CONTROLLERS IN SYSTEMS WITH RANDOM ATTRIBUTES

I. STATEMENT OF THE PROBLEM, METHOD OF SOLVING

N. N. Krasovskii and E. A. Lidskii

(Sverdlovsk)

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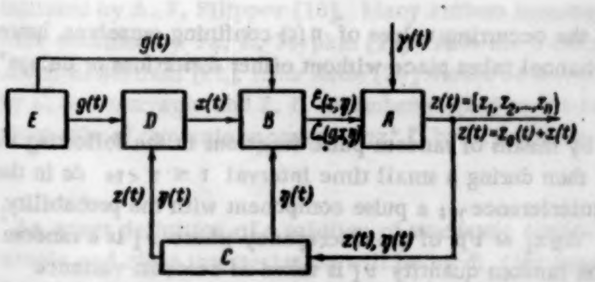
The systems are considered which are undergoing random variations during the control process and also are submitted to random interference. The method is studied of determining the controlling rule such that it ensures stability and also minimizes a given integral criterion of quality. The formulation of the problem as well as the approach to its solution are given in Part I, the latter being based on the method of the Lyapunov function [1, 2] brought up to date to comply with the principles of dynamic programming [3, 4].

1. Preliminary Notes

1. Consider a system under control (see the diagram). Here z(t) is the regulating vector quantity, $z_0(t)$ its prescribed value, $x(t) = z(t) - z_0(t)$ the discrepancy vector, g(t) the given input to the controlled device, $(\zeta + \xi)$ the output of the regulator, $\gamma(t)$ the interference, $\eta(t)$ the factor describing the random variations of the controlled object A.

The system consisting of the controlled object A and the regulator B should produce the prescribed motion $z_0(t)$ when the given signal g(t) is acting on it. In the absence of the discrepancy $[x(t) = 0]$, this is effected by the action ζ of the regulator.

The construction of the action ζ such that it would bring about the motion $z_0(t)$ prescribed in advance is outside the scope of the present paper. We shall just consider the quantity ζ as known. If some coordinates x_i of the vector x(t) do not vanish, then the member B will, in addition, generate a stabilizing signal ξ regulating the transient process. The analytic design of the quantity ξ [5] will also be our aim in this work. The rule of control $\xi(x, \eta)$ shall be determined via minimizing a given integral criterion of quality.



We shall assume that the equations of the transient process in the system can be written in the form

$$\frac{dx_i}{dt} = \varphi_i[x_1, \ldots, x_n, \eta(t), \xi] + \gamma_i, \tag{1.1}$$

$$\xi = \xi[x_1, \ldots, x_n, \eta] \tag{1.2}$$

in terms of the coordinates x_i of the discrepancy vector x(t).

The functions φ_i are assumed known; they are continuous. The fact that the main part of the system, the member A, can be subjected to random variations during the regulation process is a special characteristic of this system. This is taken into account by introducing a random variable $\eta(t)$ as one of the arguments of the functions φ_i. The statistical properties of the random component $\eta(t)$ and of the interference $\gamma(t)$ are described in section 2.

It follows directly from the definition of the quantities x(t) and $\xi(x, \eta)$ that for the prescribed motion $z_0(t)$ the equalities $x = 0$ and $\xi = 0$ are fulfilled. If the discrepancy x(t) does not vanish at the initial time instant $t = t_0$,

then a transient process takes place in the system, the process being described by the stochastic equations (1.1); the member B produces the signal ξ following the measuring of the values of $\eta(t)$ and $x(t)$ which have occurred, the member B working as an ideal regulator [6] in accordance with the equation (1.2). This process is a probabilistic one [7].

2. Properties of Random Functions $\eta(t)$ and $\gamma(t)$

The behavior of the object at the time instant t is determined by the values which are actually being taken by the random function $\eta(t)$. An object with electrical resistances can serve as an example, the resistances being functions of a randomly varying temperature $\eta(t)$. It shall be assumed that $\eta(t)$ is a Markovian random process [7].

The symbol $P[L / Q]$ denotes the conditional probability of the event L when Q has taken place, the symbol $M\{l / Q\}$ denotes the conditional expectation of the random quantity l when Q has taken place. $o(\varepsilon)$ are understood to be expressions of higher order of smallness than the small quantity ε, and $O(\varepsilon)$ a quantity of the same order of smallness as ε. The random variations $\eta(t)$ are described by means of the functions $q(\alpha)$ and $q(\alpha, \beta)$ in the following way:

$$P[\eta(\tau) = \alpha \, (t \leqslant \tau < t + \Delta t) / \eta(t) = \alpha] = 1 - q(\alpha)\,\Delta t + o(\Delta t), \tag{2.1}$$

$$P[\eta(\tau^*) \leqslant \beta, \eta(\tau^*) \neq \alpha \, (t < \tau^* < t + \Delta t) / \eta(t) = \alpha] = \\ = q(\alpha, \beta)\,\Delta t + o(\Delta t). \tag{2.2}$$

It can be seen from the definition of the functions $q(\alpha)$ and $q(\alpha, \beta)$ that $q(\alpha, -\infty) = 0$, $q(\alpha, \infty) = q(\alpha)$ are true. When $\eta(t)$ can take only one of the k values $\eta = \{\alpha_1, \ldots, \alpha_k\}$, then in order to describe fully the process, it is enough to know the transition matrix $\|p_{ij}\|$, where

$$P[\eta(t + \Delta t) = \alpha_j / \eta(t) = \alpha_i] = p_{ij}\Delta t + o(\Delta t), \tag{2.3}$$

$$q(\alpha_i) = \sum_{j \neq i}^{k} p_{ij}, \quad q(\alpha_i, \beta) = \sum_{j=1}^{m} p_{ij} \quad \text{for} \quad \alpha_m \leqslant \beta < \alpha_{m+1}.$$

If the probability density exists for the function $q(\alpha, \beta)$ that is if $q(\alpha, \beta) = \int_{-\infty}^{\beta} p(\alpha, v)\, dv$, then

$$P[\beta_1 < \eta(t + \Delta t) < \beta_2, \eta(t + \Delta t) \neq \alpha / \eta(t) = \alpha] = \Delta t \int_{\beta_1}^{\beta_2} p(\alpha, v)\, dv + o(\Delta t). \tag{2.4}$$

It shall also be assumed that it is possible to measure the occurring values of $\eta(t)$ confining ourselves, however, to the case when the transmission of the signal $\eta(t)$ in the channel takes place without either distortions or delays* until it reaches the member B (see the figure).

3. The interference γ at the input will be described by means of random pulse functions in the following way: suppose the quantities $x(t)$, $\eta(t)$ and $\xi(t)$ have taken place, then during a small time interval $t \leq \tau < t + \Delta t$ in the equation (1.1) one has to take into account having in the interference γ_i; a pulse component with the probability $p(\Delta t) = \lambda \Delta t + o(\Delta t)$; the latter produces a stepwise change $\Delta_\mu x_i \approx \nu\mu$ of the discrepancy where ν_i is a random quantity and μ_i a known function. The average value of the random quantity ν_i is taken as zero; its variance $M\{\nu_i^2\} = \sigma_i^2 \geq 0$ and the correlation coefficients k_{ij} (where $M\{\nu_i \nu_j\} = k_{ij}\sigma_i\sigma_j$) are assumed as known. We shall be considering only the limiting case of the interference described above $\lambda \to \infty$ and $\sigma_i \to 0$ in such a way that $\lambda\sigma_i^2 = \text{const}$ [9]. In the third part of this work when considering linear systems, the assumption of the limiting case $\lambda \to \infty$ and $\sigma_i \to 0$ shall be dropped.

* Should there be distortions in the quantity $\eta(t)$ during its transmission to the block B, this will not involve any essential complications provided that the distortions at different time instants are not correlated. In fact, if in such a case the distortion $\eta \to \eta^*$ is described by the distribution function $F[\beta / \alpha] = P[\eta^* \leqslant \beta / \eta(t) = \alpha]$, one can obtain from the law of large numbers [7] that if the control rule is taken as $\xi = \xi^*[x, \eta^*]$, the process will proceed as if there were no distortion but with a control rule $\xi(x, \eta) = M[\xi^*(x, \eta^*) / \eta] = \int_{-\infty}^{\infty} \xi^*(x, v)\, d_v F(v, \eta)$.

The latter integral should be regarded as Stieltjes integral [8].

3. One can verify* that with random functions η and γ as described above, a Markov random process [7] takes place in the system (1.1)-(1.2), the process occurring in the phase space $\{x_1, \ldots, x_n, \eta\}$. This signifies that the initial conditions $x_{10}, \ldots, x_{n0}, \eta_0, t_0$ determine the random functions $x_i(t) = x_i(x_0, \eta_0, t_0, t)$ $(i = 1, \ldots, n)$, $\eta(t) = \eta(\eta_0, t_0, t)$ $(t \geq t_0)$, for $t > t_0$, such that the probability distribution of the quantities $\{x_i(t)\}$, $\eta(t)$ is not affected by the knowledge of the actual behavior of the random process for $\tau < t_0$.

3. Formulation of the Problem

1. Let a function $\omega[x_1, \ldots, x_n, \eta, \xi]$ be given, positive definite in the x_i and such that a criterion of quality of the transient process is defined with its acid

$$I_\xi[x_0, \eta_0] = \int_0^\infty M\,(\omega/x_1 = x_{10}, \ldots, x_n = x_{n0},\ \eta = \eta_0 \ \text{ for } \ t = t_0 = 0)\,dt, \tag{3.1}$$

where

$$\omega = \omega[x_1(t), \ldots, x_n(t), \eta(t), \xi(x(t), \eta(t))].$$

The problem can be formulated in the following way. It is required to find the function $\xi = \xi^0(x_1, \ldots, x_n, \eta)$ such that if x(t) is the solution of the system (1.1)-(1.2), the following conditions must be satisfied: a) the given motion $x = 0$ is stable in probability [12]; b) the deviations $\{x_{i0}\}$ which occur in the system will generate a process asymptotically stable in the probability [12] with respect to $x = 0$; c) the integral (3.1) attains for a selected control rule $\xi = \xi^0(x, \eta)$ its minimum value (for all admissible initial conditions x_0, η_0) that is it is less than for any other choice of ξ (from a certain well defined family of functions $\{\xi(x, \eta)\}$).

Note. The problem is only formulated when the intensity of the interference γ approaches zero with the discrepancy x vanishing. When this condition does not take place, the process can persist in the neighborhood of the given motion $x = 0$ with an accuracy of an avoidable error $\Delta > 0$ $(\Sigma x_i^2(t) < \Delta^2)$. In this case the conditions a) and b) should be valid when $\varepsilon > \Delta$ and the average value $M\{\omega\}$ under the integral sign in (3.1) must now be computed for realizations $x^{(p)}(t)$ such that $\Sigma x_i^2(\tau) > \Delta^2$ when $t_0 \leq \tau < t$.

2. The problem under consideration belongs to the class of optimum control problems. The optimum selection of parameters in linear systems based on the Lyapunov functions method was investigated by N. G. Chetaev [2, 13]. The problem of least action time was formulated, and the first results given by A. A. Fel'dbaum [14]. The same problems were investigated by A. Ya. Lerner in a number of papers (see, for instance,[15]). Some very deep results in the mathematical theory of optimum control are due to L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko [16] whose investigations became the basis for a considerable number of studies (see the studies of L. I. Rozonoér [17]). Fundamental theoretical inquiries with regard to the existence of optimal solutions were initiated by A. F. Filippov [18]. Many authors investigated the sampled-data systems. Interesting results in this field were obtained by Ya. Z. Tsypkin [19]. Also the publications of R. Kulikowski [20], R. Bellman [21], R. E. Kalman and J. E. Bertram [22], J. La Salle [23] should be noted. The statistical aspects of optimum control were examined by L. S. Pontryagin and E. F. Mishchenko [24] on the basis of the maximum principle, and on the basis of the maximum rule of dynamic programming** by R. Bellman and his colleagues [4].

* An exact definition of a solution of stochastic equations (1.1) and (1.2) shall not be given here, only the following simple and clear interpretation will be used. One assumes that to each realization $\eta^{(p)}(t)$ and $\gamma^{(p)}(t)$ of the random functions $\eta(t)$ and $\gamma(t)$ there corresponds the realization of the solution x(t) satisfying the equations (1.1) and (1.2), with $\eta = \eta^{(p)}(t)$, within the intervals $t_k \leq t < t_{k+1}$, which are free from pulses entering the realization of $\gamma^{(p)}(t)$; at the instant $t = t_k$ of the pulse occurring, the realization of $x^{(p)}{}_i(t)$ suffers a jumplike change $\Delta_\mu x_i^{(p)} = v_{i\mu_i}(x, \xi, \eta)$. By giving a full and exact development to the points 1-3 of the section 2 it is possible by using the terminology of the theory of random processes [7-11] to verify the existence of a random process x(t) which is a solution of the equations (1.1)-(1.2) in an exact meaning of the word.

** In the latter paper the problems have certain characteristics which make them different from the usual problems of optimum filtering, prediction, etc., explored in [25-27]; they also differ from the statistical problems considered for instance in the papers [28-29].

The present work deals mainly with the studies of the problem formulated in para. 1 of this section, and it is based on the method of the Lyapunov functions [30-31] and on the use of methods of dynamic programming. In Part 2 we shall describe how to find approximations to optimum functions and how to solve the problem. This procedure shall be based on a continuous deformation of the system via the introduction of a parameter [30]. In Part 3 a solution is given to the problem of minimum squared error in the case of linear systems. Studies of the optimal integral criteria of quality in linear systems were initiated by A. A. Krasovskii and A. A. Fel'dbaum (see [32] and the literature there). The problem investigated by A. M. Letov in its deterministic form is now generalized by the approach adopted in our work.

4. General Approach to the Method of Solution

We shall assume that a function $v(x_1, \ldots, x_n, \eta)$ has been found to be such that it will satisfy the following conditions*: a) the function $v(x_1, \ldots, x_n, \eta)$ is positive definite for all $-\infty < x_i < \infty$ $(i = 1, \ldots, n)$, that is there exists a continuous function $w(x_1, \ldots, x_n)$ such that $v(x_1, \ldots, x_n, \eta) \geq w(x_1, \ldots, x_n) > 0$ for all η and $x \neq 0$, and also $\lim w(x_1, \ldots, x_n) = \infty$ if $\Sigma x_i^2 \to \infty$; b) the function $v(x, \eta)$ admits a higher limit,** that is $v(x_1, \ldots, x_n, \eta) \leq W(x_1, \ldots, x_n)$, where the continuous function W satisfies the equality $W(0, \ldots, 0) = 0$; c) the generalized derivative [12, 31] $(dM\{v\}/dt)_{\xi^0}$ when (1.1) and (1.2) are valid, satisfies for $\xi = \xi^0(x, \eta)$ the conditions $|dM\{v\}/dt| \geqslant vk$ $(k = \text{const})$ and also

$$\left(\frac{dM\{v\}}{dt}\right)_{\xi^0} + \omega\,[x, \eta, \xi^0] = 0, \tag{4.1}$$

with the left-hand side attaining its minimum when $\xi = \xi^0$, that is

$$\left(\frac{dM\{v\}}{dt}\right)_{\xi^0} + \omega\,[x, \eta, \xi^0] = \min_{\xi}\left[\left(\frac{dM\{v\}}{dt}\right)_{\xi} + \omega\,(x, \eta, \xi)\right]. \tag{4.2}$$

Remark. In the terminology of the theory of random processes the derivative $dM\{V\}/dt$ is determined by the infinitely small differential operator [10] of the Markov process $x(t), \eta(t)$. This can be illustrated in the following way. Suppose the values $x(t) = x$, $\eta(t) = \eta$ were realized. From the point (x, η) there originates a pencil of trajectories $\{x(\tau), \eta(t)\}$ $(\tau \geq t)$, generating the random quantity

$$[v\,(x\,(t + \Delta t), \eta\,(t + \Delta t))\,/\,x\,(t) = x, \eta\,(t) = \eta].$$

In order to compute the derivative $dM\{v\}/dt$ we evaluate the mathematical expectation $M\ \{v\,(x\,(t + \Delta t), \eta\,(t + \Delta t))\,/\,x\,(t) = x, \eta\,(t) = \eta\}$, from which we compute $v(x, \eta)$, divide the result by $\Delta t > 0$ and proceed to the limit with $\Delta t \to 0$.

The function $v^0(x, \eta)$ satisfying the conditions a) - c) shall be called an optimum Lyapunov function. From the results of the theory of dynamic programming [3-4] and from the theorems on stability in probability [12], it follows that the function $v^0(x, \eta)$ together with the corresponding function $\xi^0(x, \eta)$ from the formulas (4.1) and (4.2) provide a solution of our problem,*** that is when $\xi = \xi^0(x, \eta)$ all the requirements a) - c) of the section 3 are fulfilled. Thus the functions v^0 and ξ^0 satisfying the conditions a) - c) solve our problem. The actual construction of the functions v^0 and ξ^0 can be found in Part 2 and Part 3.

The authors wish to express their gratitude to A. M. Letov for discussing the present work with them.

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* Any initial deviations x_0 are assumed permissible. If deviations in some domain G are only allowed, one has to modify the subsequent considerations accordingly.

** A quadratic form $v = \Sigma b_{ij}x_ix_j$ satisfies, for example, the conditions a) and b) if its coefficients are bounded and if they satisfy the Sylvester's criterion of constant sign, that is $b_{11}(\eta) > \varepsilon, |b_{ij}(\eta)|_1^2 > \varepsilon, \ldots, |b_{ij}(\eta)|_1^n > \varepsilon$, $\varepsilon = \text{const} \geq 0$, uniformly in η.

*** In [12] stochastic systems were considered of a more specialized type, nevertheless the theorems on stability in probability remain also valid in the case now being considered by us. For the unavoidable error $\Delta > 0$ (see the Note in section 3) the requirements a) - c) should be modified for the domain $\Sigma x_i^2 > \Delta^2$.

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THE PASSAGE OF RANDOM SIGNALS THROUGH A TIME DISCRIMINATOR AND AN INTEGRATING AMPLIFIER

I. FORMULATION OF A RECURSION RELATIONSHIP FOR DETERMINING THE COORDINATE LATTICE FUNCTIONS THAT CHARACTERIZE RANDOM PROCESSES IN A PULSE SYSTEM

F. M. Kilin

(Leningrad)

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The paper analyzes the passage of random signals through a time discriminator and an integrating amplifier. Based on the analysis performed in this paper, the properties of a pulse system are determined and their relationship to the properties of asynchronous detector are demonstrated. For a pulse system we find the equivalent circuits with constant parameters in which the processes are continuous. Replacing the pulse system with an equivalent circuit simplifies the analysis and leads to simple expressions for the characteristics of the original system. These expressions take into account the special features of operation of the pulse system with a thoroughness that is adequate for practical purposes.

In part I of this paper we derive the recursion relationship in matrix form for determining the coordinate lattice functions which characterize the random processes in the pulse system.

Introduction

A pulse system consisting of a time discriminator and an operational amplifier plays an important part in automatic radar units and radionavigation systems. In particular, such a system is an important element in an automatic range finding unit for which certain theoretical problems were analyzed in [1-5] et al. The use of frequency methods in analyzing a linear pulse control system with time selectors was studied in [6].

The engineering principles of designing time discriminators and certain characteristics of time discriminators have been dealt with in a number of papers. Individual sets of data on these problems are cited in the books [7-10] et al.* Our literature does not yet contain papers in which a) the scattered data on time discriminators is systematized and generalized, and b) a complete theory for the operation of these devices in various automatic systems is presented.

In order to clarify the properties of the pulse system under study when random signals pass through it we make use of the paper by V. S. Pugachev on the canonical expansion of random functions [13]. The use of other methods for analyzing random processes in a pulse system with time selectors gives rise to difficulties, since these processes are nonstationary by their internal nature. However, these nonstationary processes can be described by a system of stationary lattice functions. These provide the possibility of finding equivalent stationary continuous systems that approximately simulate the dynamics of a pulse system with time selectors.

Based on this we developed a method for analyzing the dynamics of pulse systems with time selectors which is presented in [14, 15]. The application of this method made it possible to discover many important details which are characteristic of the operation of pulse systems with time selectors. The conversion of signals in pulse systems with a time discriminator, as we shall demonstrate below, has much in common with conversionin a synchronous detector.

* Cf. also [11] and [12].

Problems of synchronous detection, which is also called selective detection, were first studied in [16]. Synchronous detection usually is applied in those cases where higher noise stability requirements are imposed on a radio-engineering system. For example, it is widely used in experimental radiospectroscopy units [17, 18]. Synchronous detection in these units assures a high sensitivity of the electronic devices. In radioastronomy the combination of a highly stable synchronous detector and a narrow-band output unit assures isolation of very weak signals whose power is only thousandths of the noise power [19, 20]. At the present time many forms of synchronous detection are known which are applied in various automatic systems. In this regard the theory of synchronous detection requires further development and refinement.

We should, however, pay attention to the fact that although the conversion of random signals in time discriminators and an integrating amplifier has much in common with conversion in a synchronous detector, there are substantial differences between these types of conversion; we shall demonstrate these differences in this paper.

The transient responses in a pulse system consisting of a time discriminator and an integrating amplifier are characterized by large time constants; i.e., the specified system has a sufficiently large inertia. Therefore only the low-frequency components of the input signal can have an effect on the operation of such a system. In solving applied problems and performing engineering computations we usually originate from this fact and replace the pulse system with an equivalent continuous system which passes the low-frequency components of the input signal.* In reality we encounter the opposite picture. Due to the presence of a time discriminator in the system the high-frequency components of the input signal produce the principal effect on the system; this contradicts the statement made above. In fact, there is no contradiction here. This can be explained physically on the basis of the fact that the high-frequency components are converted into low-frequency components in the time discriminator; these latter components cannot be filtered by the narrow-band filter. Here frequency conversion in the system is achieved by means of linear elements. This is the principal difference between the specified system and other systems with identical transient responses.

Using a time discriminator and an integrating amplifier, we can achieve the isolation and analysis of the information which is carried in radar signals. A penetration into the essence of the physical phenomena which occur in the pulse system dealt with in this paper will make it possible to design highly effective units for isolating and analyzing radar information and transmitting it to other elements in an automatic system. If a time discriminator is included in an automatic system as a detecting element, then its properties have an appreciable effect on the operation of the automatic system as a whole. In part III of this paper we shall turn our attention a) to the design of a continuous system which is equivalent to the pulse system, and b) to the derivation of approximate expressions which are simple in structure but consider the specifics of the pulse system in a sufficiently complete manner.

1. The Mathematical Description of the Dynamics of the Pulse System

In order to determine the basic properties of the pulse system discussed above we shall limit ourselves to the solution of the linear problem. In accordance with this we shall study a system consisting of a time discriminator and an inertial section in which the processes can be described by the equations

$$[\tau_1(t)D+1]U_1(t)=K_0\gamma(t)[E_s\psi_0(t)+E_n(t)], \qquad (1.1)$$
$$[\tau_2 D+1]U_2(t)=K_1U_1(t).$$

Here $D = d/dt$ is the differentiation operator, t is the "running" time, U_1 is the output signal of the time discriminator, U_2 is the output signal of the operational amplifier which is an inertial section with the time constant τ_2. (If we increase the time constant τ_2 of the inertial section, then its properties will approach the properties of an integrating section.) K_0 and K_1 are the corresponding gains.

The second equation in (1.1) describes the processes in the vacuum-tube integrator whose operator transfer function is written as

$$K_{v.i.}(D)=\frac{K_1}{(K_1+1)CRD+1},$$

*Cf. for example, [21].

where R is the resistance of the integrator input circuit, C is the capacitance of the capacitor in the feedback circuit (Fig. 1). From this equation it follows that

$$\tau_2 = (K_1 + 1)\,CR.$$

More detailed data on a vacuum-tube integrator and operational amplifiers is given in [22].

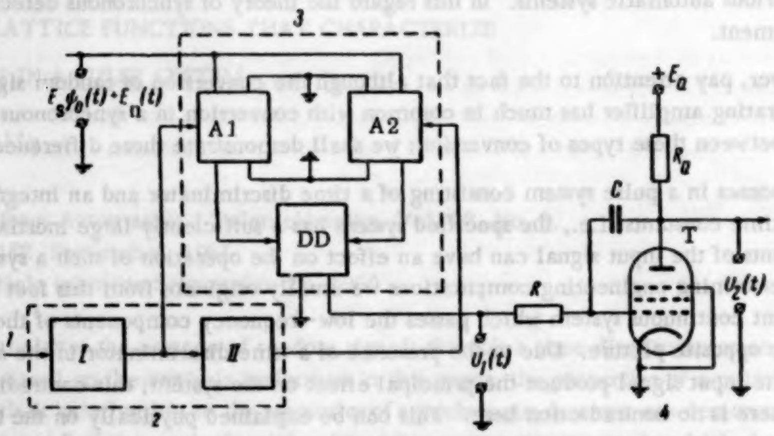

Fig. 1. 1) From starting pulse generator; 2) selector pulse generator; 3) time discriminator; 4) vacuum-tube integrator.

The circuit for the pulse system is shown in Fig. 1. Here A1 and A2 are coincidence amplifiers (time selector), DD is a difference detector. The coincidence amplifiers A1, A2 and the difference detector DD form the time discriminator circuit. Each coincidence amplifier passes only that portion of the input signal which coincides in time with the corresponding selector pulse. The selector pulse generator forms two pulse trains that are time shifted by the amount α_0 with respect to one another. The difference signal obtained at the output of the time discriminator is applied to the integrator CR based on the tube T. The operation of the time discriminator is described in greater detail in part I [5].

The useful signal $E_s\psi_0(t)$ and the noise $E_n(t)$ are applied to the input of the system. We shall denote the input signal by

$$E_{in}(t) = E_s\,\psi_0(t) + E_n(t). \tag{1.2}$$

The first equation in (1.1) describes the conversion of the input signal $E_{in}(t)$ in the time discriminator. According to this equation the input signal is first subjected to time conversion described by the pulse function $\nu(t)$. The graph of this function is shown in Fig. 2b. After time conversion the signal is subjected to additional conversion in a fourpole with a variable parameter τ_1 that varies stepwise. The second equation (1.1) describes the subsequent conversion of the signal in the inertial section with the time constant τ_2 after the signal has passed through the time discriminator.

The pulse function $\psi_0(t)$ is written as

$$\psi_0(t) = \sum_{k=0}^{\infty} [H(t - \eta_k - kT) - H(t - \alpha - \eta_k - kT)]. \tag{1.3}$$

Here α is the duration of the pulses which form the useful component, T is the repetition period for the reference pulses, H(t) is a unit switching function which satisfies the conditions

$$H(t) = \begin{cases} 0 & \text{for} \quad t < 0, \\ 1 & \text{for} \quad t \geqslant 0. \end{cases} \tag{1.4}$$

The lattice function $\eta_k (k = 0, 1, 2, 3, \ldots, \infty)$ describes the time position of the kth pulse of the useful component relative to the reference pulse with the number k.

The pulse function $\gamma(t)$ which describes the conversion of the input signal in the time discriminator can be represented by the following formulas:

$$\gamma(t) = \gamma_1(t) - \gamma_2(t). \tag{1.5}$$

$$\gamma_1(t) = \sum_{k=0}^{\infty} [H(t - \vartheta_k - kT) - H(t - \alpha_0 - \vartheta_k - kT)],$$

$$\gamma_2(t) = \sum_{k=0}^{\infty} [H(t - \alpha_0 - \vartheta_k - kT) - H(t - 2\alpha_0 - \vartheta_k - kT)]. \tag{1.6}$$

Here α_0 is the duration of the tracking (selector) pulse. The lattice function $\vartheta_k (k = 0, 1, 2, \ldots, \infty)$ determines the time displacements of the tracking pulses relative to the reference pulses. The tracking error in the case under study is determined from the equation

$$\Delta\alpha[n] = \eta_n + \frac{\alpha}{2} - \vartheta_n - \alpha_0. \tag{1.7}$$

We note the fact that in our papers [5] the pulse functions $\psi_0(t)$, $\gamma_1(t)$ and $\gamma_2(t)$ are written in a different form. The difference is very slight and consists of the fact that the time origin in this present paper is chosen in such a way that it coincides with the leading edge of the reference pulse with the number 0. The form of notation for (1.3) and (1.6) proves to be more convenient in studying random processes in the dynamic system under study here.

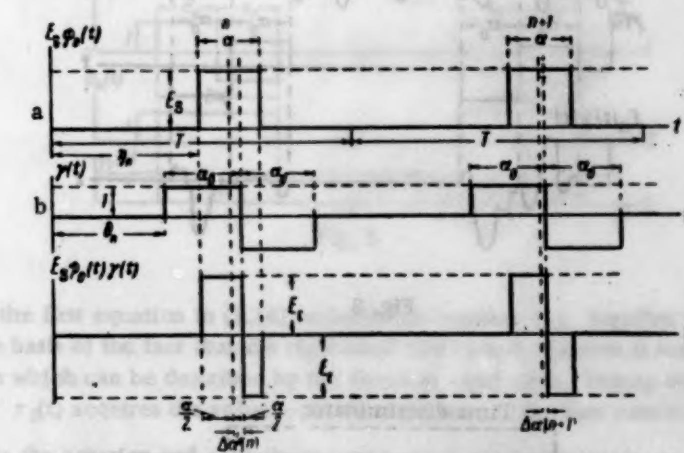

Fig. 2

Figure 2 shows graphs of the pulse functions $E_s\psi_0(t)$, $\gamma(t)$ and $E_s\psi_0(t)\gamma(t)$. The quantity ϑ_n determines the position of the leading edge of the tracking pulse I with the number n relative to the reference pulse with the same number. Analogously, the quantity η_n determines the position of the useful-component pulse with the number n relative to the corresponding reference pulse.

Figure 3 shows the graph of one realization of the random function $E_n(t)$ which describes the noise at the input of the time discriminator. The same figure shows the graph of the function $E_n(t)\gamma(t)$ which expresses the transformation of the specified realization of the random process $E_n(t)$ by the system of time selectors (coincidence amplifiers).

Figure 4 shows the block diagram of the pulse system. The time discriminator is represented by 2 serially connected fourpoles which are inscribed in the dotted rectangle. The conversion of the input signals in the time selector and the difference stage is represented by a fourpole with a variable gain $K_{0\nu}(t)$. The subsequent gain of the signal in the circuits with varying parameters is represented by a fourpole with the operator transfer function

$$Q_1(D,\ t) = \frac{1}{\tau_1(t)\,D + 1}\ . \tag{1.8}$$

Since a continuous signal consisting of the useful component and the noise appears at the input, it follows that the parameters of the time discriminator will vary in accordance with the application of the selector pulses to the system; i.e.,

$$\tau_1(t) = \tau_{10}\,\varphi_0(t) + \tau_{11}\varphi_1(t), \tag{1.9}$$

where

$$\varphi_0(t) = \gamma_1(t) + \gamma_2(t), \quad \varphi_1(t) = H(t) - \varphi(t). \tag{1.10}$$

In expanded form the pulse function is written as

$$\varphi(t) = \sum_{k=0}^{\infty} [H(t - \vartheta_k - kT) - H(t - 2a_0 - \vartheta_k - kT)]. \tag{1.11}$$

Figure 5 shows graphs of the pulse functions $\gamma(t)$, $\varphi_0(t)$ and $\varphi_1(t)$.

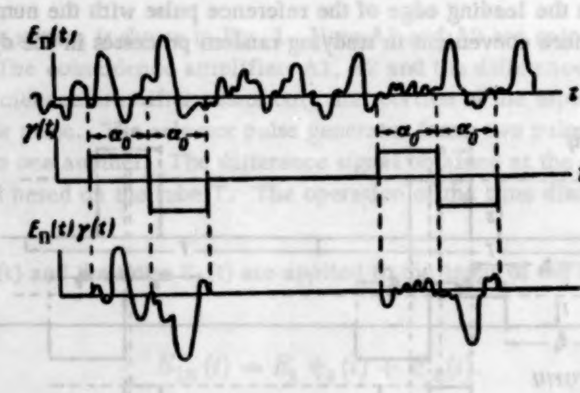

Fig. 3

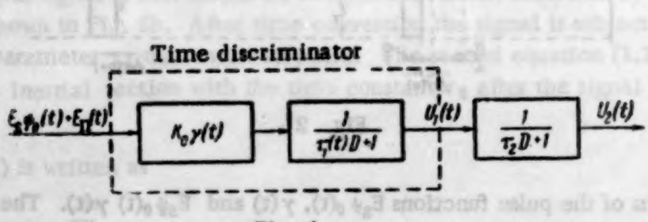

Fig. 4

We shall assume that the noise component $E_n(t)$ is described by a stationary random function for which the integral canonical form is written as*

* Cf. [13], pp. 315-323.

$$E_n(t) = m_e + \int_{-\infty}^{+\infty} Y_e(\omega)\, e^{i\omega t} d\omega,$$

$$k_e(t - t') = \int_{-\infty}^{+\infty} S_e(\omega)\, e^{i\omega t}\, \overline{e^{i\omega t'}}\, d\omega, \tag{1.12}$$

$$K_2(\omega, \omega') = M\,[Y_e(\omega)\, \overline{Y_e(\omega')}] = S_e(\omega)\, \delta(\omega - \omega').$$

Here $k_e(t-t')$ and $S_e(\omega)$ respectively denote the correlation function and the spectral density of the noise $E_n(t)$. The constant m_e is not passed by the time discriminator; the output signal which it produces is equal to zero. Therefore, it will not be a simplification if we assume that the component m_e in the first of Eq. (1.12) will be equal to zero; i.e.,

$$m_e = M\,[E_n(t)] = 0. \tag{1.13}$$

We shall divide both sides of the first equation in (1.1) by $\tau_1(t)$ and the second equation by τ_2. As a result we will obtain:

$$[D + a_{11}(t)]\, U_1(t) = \frac{K_0\gamma(t)}{\tau_{10}}\,[E_s\psi_0(t) + E_n(t)],$$

$$a_{21}U_1(t) + (D + a_{22})\, U_2(t) = 0. \tag{1.14}$$

Here

$$a_{11}(t) = \frac{1}{\tau_1(t)}, \quad a_{22} = \frac{1}{\tau_2}, \quad a_{21} = -\frac{K_1}{\tau_2}. \tag{1.15}$$

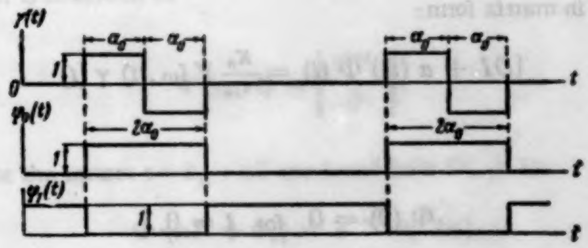

Fig. 5

The right side of the first equation in (1.14) includes the constant τ_{10} together with the variable $\tau_1(t)$. This can be explained on the basis of the fact that the right hand side of this equation is non-zero only over time intervals during which pulses which can be described by the function $\varphi_0(t)$ exist. During the intervals between pulses $\varphi_0(t)$ when the constant $\tau_1(t)$ acquires the value $\varphi_0(t)$ the right side of the first equation in (1.14) goes to zero.

In order to abridge the notation and achieve compactness in our presentation we shall write the equations in matrix form

$$[DI + a(t)]\, U(t) = \frac{K_0\gamma(t)}{\tau_{10}}\,[f(t) + E(t)]. \tag{1.16}$$

Here

$$a(t) = \begin{Vmatrix} a_{11}(t), & 0 \\ a_{21}, & a_{22} \end{Vmatrix}, \qquad U(t) = \begin{Vmatrix} U_1(t) \\ U_2(t) \end{Vmatrix}, \tag{1.17}$$

$$f(t) = \begin{Vmatrix} E_s\psi_0(t) \\ 0 \end{Vmatrix}, \qquad E(t) = \begin{Vmatrix} E_n(t) \\ 0 \end{Vmatrix}. \tag{1.18}$$

The quadratic matrix a(t) can also be written as

$$a(t) = a^0\varphi_0(t) + a^1\varphi_1(t), \tag{1.19}$$

where

$$a^0 = \begin{Vmatrix} a_{11}^0, & 0 \\ a_{21}, & a_{22}, \end{Vmatrix}, \qquad a^1 = \begin{Vmatrix} a_{11}^1, & 0 \\ a_{21}, & a_{22} \end{Vmatrix}. \tag{1.20}$$

The coefficients a_{11}^0 and a_{11}^1 are defined in terms of the time constants τ_{10} and τ_{11} using the formulas

$$a_{11}^0 = \frac{1}{\tau_{10}}, \quad a_{11}^1 = \frac{1}{\tau_{11}}. \tag{1.21}$$

2. Equations for the Coordinate Functions which Characterize the Random Process in the Time Discriminator and the Operational Amplifier*

We introduce the coordinate functions for the time discriminator and the operational amplifier: $\Phi_1(t)$ and $\Phi_2(t)$, respectively. Based on (1.14) the equations for $\Phi_1(t)$ and $\Phi_2(t)$ are written as

$$\begin{gathered}[D + a_{11}(t)]\,\Phi_1(t) = \frac{K_0}{\tau_{10}}\gamma(t)\,e^{i\omega t}, \\ a_{21}\Phi_1(t) + [D + a_{22}]\,\Phi_2(t) = 0. \end{gathered} \tag{2.1}$$

The initial conditions for Eq. (2.1) are written as

$$\Phi_1(0) = \Phi_2(0) = 0 \text{ for } t = 0. \tag{2.2}$$

We shall write Eq. (2.1) in matrix form:

$$[DI + a(t)]\,\Phi(t) = \frac{K_0}{\tau_{10}}Y(\omega, t)\,\gamma(t). \tag{2.3}$$

The initial conditions are

$$\Phi(0) = 0 \quad \text{for} \quad t = 0. \tag{2.4}$$

Here

$$\Phi(t) = \begin{Vmatrix} \Phi_1(t) \\ \Phi_2(t) \end{Vmatrix}, \tag{2.5}$$

$$Y(\omega, t) = \begin{Vmatrix} e^{i\omega t} \\ 0 \end{Vmatrix}. \tag{2.6}$$

We shall find the solution for Eq. (2.3) over the time interval

$$nT \leqslant t \leqslant (n+1)\,T. \tag{2.7}$$

The time interval (2.7) is subdivided into three regions:

$$\begin{aligned} &\text{Region} \quad \text{I}: nT < t \leqslant nT + \vartheta_n, \\ &\text{Region} \quad \text{II}: nT + \vartheta_n < t \leqslant nT + \vartheta_n + 2a_0, \\ &\text{Region} \quad \text{III}: nT + \vartheta_n + 2a_0 < t \leqslant (n+1)\,T. \end{aligned} \tag{2.8}$$

* The general method for determining the coordinate functions for pulse systems and its application to individual particular cases was presented in detail in [15].

We shall find the expression for the matrix function $\Phi(t)$ in each region.

3. Determining the Coordinate Function $\Phi(t)$ in Region I

Equation (2.3) in region I is written as

$$[DI + a^1]\,\Phi(t) = 0, \quad nT < t \leqslant nT + \vartheta_n. \tag{3.1}$$

We shall write the initial conditions for the instant t = nT:

$$\Phi[n] = \Phi(nT). \tag{3.2}$$

Solving Eq. (3.1) and taking into account the initial conditions (3.2), we obtain

$$\Phi(t) = e^{-a^1(t-nT)}\Phi[n], \qquad nT < t \leqslant nT + \vartheta_n. \tag{3.3}$$

4. Determining the Coordinate Function $\Phi(t)$ in Region II

We shall write Eq. (3.2) for the region II:

$$[DI + a^0]\,\Phi(t) = \frac{K_0}{\tau_{10}} Y(\omega, t)\,[H(t - \vartheta_n - nT) - 2H(t - \alpha_0 - \vartheta_n - nT)], \qquad nT + \vartheta_n < t \leqslant nT + \vartheta_n + 2\alpha_0. \tag{4.1}$$

The matrix column $Y(\omega, t)$ is written as

$$Y(\omega, t) = \begin{Vmatrix} e^{i\omega t} \\ 0 \end{Vmatrix}. \tag{4.2}$$

The initial conditions for the instant $t = \vartheta_n + nT$ are found from Eq. (3.3):

$$\Phi(nT + \vartheta_n) = e^{-a^1\vartheta_n}\Phi[n]. \tag{4.3}$$

The solution of Eq. (4.1) for the initial conditions (4.3) is written as follows:

$$\Phi(t) = e^{-a^0(t-\vartheta_n-nT)}e^{-a^1\vartheta_n}\Phi[n] + W(t), \qquad nT + \vartheta_n < t \leqslant nT + \vartheta_n + 2\alpha_0. \tag{4.4}$$

The function

$$W(t) = \begin{Vmatrix} W_1(t) \\ W_2(t) \end{Vmatrix} \tag{4.5}$$

is the response of the pulse system to the input signal $Y(\omega, t)$. This function satisfies the equation

$$[DI + a^0]\,W(t) = \frac{K_n Y(\omega, t)}{\tau_{10}}\,[H(t - \vartheta_n - nT) - 2H(t - \alpha_0 - \vartheta_n - nT)], \qquad nT + \vartheta_n < t \leqslant nT + \vartheta_n + 2\alpha_0 \tag{4.6}$$

for the initial conditions

$$W(nT + \vartheta_n) = 0. \tag{4.7}$$

In order to solve Eq. (4.6) for the initial conditions (4.7) we shall write the function Y(ω, t) in a different form:

$$Y(\omega, t) = \begin{Vmatrix} e^{i\omega t} \\ 0 \end{Vmatrix} = e^{i\Omega t} Y_0, \tag{4.8}$$

where

$$e^{i\Omega t} = \begin{Vmatrix} e^{i\omega t}, & 0 \\ 0, & e^{i\omega t} \end{Vmatrix}. \tag{4.9}$$

$$Y_0 = \begin{Vmatrix} 1 \\ 0 \end{Vmatrix}. \tag{4.10}$$

We shall rewrite Eq. (4.6) in new variables:

$$[DI + a^0]\, W(t) = \frac{K_0}{\tau_{10}} e^{i\Omega t} Y_0 \,[H(t - \vartheta_n - nT) - 2H(t - \alpha_0 - \vartheta_n - nT)], \quad nT + \vartheta_n < t \leqslant nT + \vartheta_n + 2\alpha_0. \tag{4.11}$$

The solution of Eq. (4.11) for the initial conditions (4.7) is written as

$$W(t) = \frac{K_0 e^{i\Omega T n} e^{i\Omega \vartheta_n}}{\tau_{10}(i\Omega + a^0)} \{[e^{i\Omega(t - \vartheta_n - nT)} - e^{-a^0(t - \vartheta_n - nT)}] - 2e^{i\Omega\alpha_0}[e^{i\Omega(t - \alpha_0 - \vartheta_n - nT)} - e^{-a^0(t - \alpha_0 - \vartheta_n - nT)}]\} Y_0 \tag{4.12}$$

or

$$W(t) = e^{i\Omega T n} e^{i\Omega\vartheta_n} Z(t), \tag{4.13}$$

where

$$Z(t) = \frac{K_0}{\tau_{10}(i\Omega + a^0)} \{[e^{i\Omega(t - \vartheta_n - nT)} - e^{-a^0(t - \vartheta_n - nT)}] - 2e^{i\Omega\alpha_0}[e^{i\Omega(t - \alpha_0 - \vartheta_n - nT)} - e^{-a^0(t - \alpha_0 - \vartheta_n - nT)}]\} Y_0. \tag{4.14}$$

5. Determining the Coordinate Function Φ(t) in the Region III

We shall write Eq. (2,3) for the region III:

$$[DI + a^1]\, \Phi(t) = 0, \; nT + \vartheta_n + 2\alpha_0 < t \leqslant (n + 1)\, T. \tag{5.1}$$

This equation coincides in form with Eq. (3.1) for region I, but it differs from it in its initial conditions which for Eq. (5.1) are written as

$$\Phi(nT + \vartheta_n + 2\alpha_0) = e^{-2a^1\alpha_0} e^{-a^1\vartheta_n} \Phi[n] + e^{i\omega(nT + \vartheta_n)} \begin{Vmatrix} Z_1(nT + \vartheta_n + 2\alpha_0) \\ Z_2(nT + \vartheta_n + 2\alpha_0) \end{Vmatrix}. \tag{5.2}$$

The quantities $Z_1(nT + \vartheta_n + 2\alpha_0)$ and $Z_2(nT + \vartheta_n + 2\alpha_0)$ depend on the parameter $i\omega$ in accordance with (4.14) and (4.9). Thus, we can introduce the substitutions:

$$R_1(i\omega) = Z_1(nT + \vartheta_n + 2\alpha_0), \; R_2(i\omega) = Z_2(nT + \vartheta_n + 2\alpha_0) \tag{5.3}$$

and

$$R(i\omega) = \begin{Vmatrix} R_1(i\omega) \\ R_2(i\omega) \end{Vmatrix}. \tag{5.4}$$

We shall write the expression for the initial conditions (5.2) in terms of the new substitutions:

$$\Phi(nT + \vartheta_n + 2\alpha_0) = e^{-2a^0\alpha_0} e^{-a^1\vartheta_n} \Phi[n] + e^{i\omega(nT+\vartheta_n)} R(i\omega). \tag{5.5}$$

We shall find a solution of Eq. (5.1) for the initial conditions (5.5)

$$\begin{gathered} \Phi(t) = e^{-a^1(t-2\alpha_0-\vartheta_n-nT)} e^{-2a^0\alpha_0} e^{-a^1\vartheta_n} \Phi[n] + e^{i\omega(nT+\vartheta_n)} e^{-a^1(t-2\alpha_0-\vartheta_n-nT)} R(i\omega), \\ nT + \vartheta_n + 2\alpha_0 < t \leqslant (n+1)T. \end{gathered} \tag{5.6}$$

6. The Recursion Relationship for Determining the Coordinate Lattice Function which Characterizes the Random Processes in a Time Discriminator and an Integrating Amplifier

Assuming that in Eq. (5.6) we have $t = (n+1)T$ and writing $\Phi[n+1] = \Phi(nT + T)$,

$$e^{-C_nT} = e^{-a^1(T-2\alpha_0-\vartheta_n)} e^{-2a^0\alpha_0} e^{-a^1\vartheta_n}, \tag{6.1}$$

we obtain the recursion relationship

$$\Phi[n+1] = e^{-C_nT}\Phi[n] + e^{i\omega(nT+\vartheta_n)} e^{-a^1(T-2\alpha_0-\vartheta_n)} R(i\omega), \tag{6.2}$$

from which it is possible to find the coordinate lattice function $\Phi[n]$.

The recursion relationship (6.2) can be simplified on the basis of the following concepts. For a time discriminator and an operational amplifier the following conditions are always satisfied:

$$|2a^0_{jk}\alpha_0| \ll 1, \qquad |a^1_{jk}T| \ll 1, \tag{6.3}$$

where the vertical lines denote the absolute values of the following products: a) the elements of matrix a^0 multiplied by the duration $2\alpha_0$ of the tracking pulses, and b) the elements of the matrix a^1 multiplied by the repetition period T of the reference pulses.

When inequalities (6.3) are satisfied the following relationships are valid:

$$\begin{aligned} e^{-2a^0\alpha_0} &\approx I - 2a^0\alpha_0, \\ e^{-a^1\vartheta_n} &\approx I - a^1\vartheta_n, \\ e^{-a^1(T-\vartheta_n-2\alpha_0)} &\approx I - a^1(T - \vartheta_n - 2\alpha_0). \end{aligned} \tag{6.4}$$

Therefore

$$C_nT \approx aT = 2a^0\alpha_0 + a^1(T - 2\alpha_0), \tag{6.5}$$

i.e., when inequalities (6.3) are satisfied the matrix C_n ($n = 0, 1, 2, 3, \ldots, \infty$) is practically independent of the number $\underline{n}$. Taking this into account, we rewrite the recursion relationship (6.2) in the form

$$\Phi[n+1] = e^{-aT}\Phi[n] + e^{i\omega(nT+\vartheta_n)} R(i\omega). \tag{6.6}$$

The resulting recursion relationship for the coordinate lattice function $\Phi[n]$ will be used in the subsequent parts of this paper to determine the correlation functions and spectral densities corresponding to the output signals of the pulse system.

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All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. ***Some or all of this periodical literature may well be available in English translation.*** **A complete list of the cover-to-cover English translations appears at the back of this issue.**

OPTIMUM FILTER DISCRIMINATION OF TELEGRAPH SIGNALS

R. L. Stratonovich

(Moscow)

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Equations determining the a posteriori probabilities of telegraph pulse signal values for the case of "white noise" interference are derived. The article also describes the block diagrams of optimum receivers that are able to perform optimum nonlinear filter discrimination and to provide filtered-out pulse signals at the output.

The theory of Markov conditional process [1, 2] can be successfully used in many practical cases for the construction and calculation of optimum filter-discrimination systems. Some of the applications were described in [3]. The present article is concerned with the application of this theory to another important particular case, namely, to the case of the filter discrimination of telegraph signals.

A telegraph signal should be understood as a two-position pulse signal, i.e., a signal which assumes the values $s(t) = \pm a$. Various assumptions can be made with respect to the statistical properties of the pulse durations and the moments of transition from one state into another. The durations of pulses and intervals can be dependent or independent, they can have different distributions, and, in particular, they can be determined (nonrandom durations). Without making our considerations very general, we shall analyze only the case of independent durations of pulses and intervals.

We shall note, however, that different generalizations of the method are possible in the case of dependent durations of pulses and intervals that are, however, correlated in a Markov chain.

Assume that a mixture of the signal and noise $r(t) = s(t) + n(t)$, which is to be subjected to optimum processing, arrives at the receiver's input. We shall consider a disturbance $n(t)$ in the form of white Gaussian noise, which has the autocorrelation function $\overline{nn_\tau} = N\delta(\tau)$. The methods for generalizing these results for the case of correlated noise are given in the Appendix.

1. Equations for the A Posteriori Probabilities

We shall begin with the derivation of some general equations for determining the a posteriori probabilities, which we shall need later.

We shall denote by $\ldots, \tau_1, \tau_2, \ldots$ the sequential instants of time when the telegraph signal $s(t)$ passes from the state $-a$ into the state $+a$. These instants of time will be called "points of upward transition." $\ldots \sigma_1, \sigma_2, \ldots$, we shall denote the sequential instants of time of the reverse transition, i.e., the "points of downward transition." If we choose a certain conditional initial instant of time $t_0 = 0$ and if we count the transition points from this instant, we have:

$$0 < \tau_1 < \sigma_1 < \ldots < \tau_k < \sigma_k < \ldots .$$

It can be considered that this process begins with the instant of time $t = t_0$ (if necessary, the change of limits $t_0 \to -\infty$ can be performed after solving the problem).

Let $w_{pr}(\tau_1, \sigma_1, \ldots, \tau_M, \sigma_M)$ be the common a priori distribution density for a sufficiently large number of sequential transition points. If the input process $r(t)$ is observed during the time interval $0 < t < T$, then, according to the reverse probability formula, the a posteriori distribution density for the indicated transition points will be

$$w_{ps}(\tau_1, \sigma_1, \ldots, \tau_M, \sigma_M) =$$

$$= Kw_{pr}(\tau_1, \sigma_1, \ldots, \tau_M, \sigma_M) \exp\left\{-\frac{1}{2N}\int_0^T [r(t) - s(t)]^2\,dt\right\}, \tag{1}$$

where K is a constant which depends on T and r(t).

In deriving Eq. (1), we used the well-known expression for the functional of the white noise probability:

$$W[n(t)] = \exp\left\{-\frac{1}{2N}\int_0^T n^2(t)\,dt\right\}, \tag{2}$$

where n(t) = r(t)−s(t) was substituted (see, for instance, [4]).

We shall now calculate the a posteriori probability that, at the instant of time t = T, the signal will be in the upper position:

$$w_1(T) = \mathrm{P}\{s(T) = a \mid r(\tau), 0 \leqslant \tau \leqslant T\}, \tag{3}$$

and also the probability of the opposite event:

$$w_2(T) = \mathrm{P}\{s(T) = -a \mid r(\tau), 0 \leqslant \tau \leqslant T\}. \tag{4}$$

If a pulse exists at the instant of time t = T, this pulse can have different ordinal numbers. Let P_k be the probability that the kth pulse will arrive at the instant of time t = T. Then, it is obvious that

$$w_1(T) = \sum_{k=1}^{M} P_k. \tag{5}$$

Here, M is a certain sufficiently large (nonrandom) number, the exact value of which is not important for theoretical considerations, since it does not figure in the final results. The probability that the number of pulses will exceed the M value is assumed to be negligibly small.

The a posteriori probability P_k can be found from the distribution density (1) by integration with respect to all possible positions of the transition points:

$$P_k = \int_{\tau_k<T} \ldots \int_{\sigma_k>T} w_{ps}(\tau_1, \sigma_1, \ldots, \tau_M, \sigma_M)\,d\tau_1 \ldots d\sigma_M. \tag{6}$$

Therefore, according to (1) and (5), we obtain:

$$w_1(T) = K\sum_{k=1}^{M}\int_{\tau_k<T} \ldots \int_{\sigma_k>T} w_{pr}(\tau_1, \ldots, \sigma_M)\exp\left\{-\frac{1}{2N}\int_0^T (r-s)^2\,dt\right\}\times$$

$$\times\, d\tau_1, \ldots, d\sigma_M.$$

However, for $\sigma_k > T$, the integral $\int_0^T (r-s)^2\,dt$ is independent of $\tau_{k+1}, \sigma_{k+1}, \ldots, \sigma_M$.

$$\int \ldots \int w_{pr}(\tau_1, \ldots, \sigma_M)\exp\left\{-\frac{1}{2N}\int_0^T (r-s)^2\,dt\right\} d\tau_{k+1}, \ldots, d\sigma_M =$$

$$= w_{pr}(\tau_1, \ldots, \sigma_k)\exp\left\{-\frac{1}{2N}\int_0^T (r-s)^2\,dt\right\}$$

and

$$w_1(T) = K \sum_{k=1}^{M} \int_{\tau_k < T} \cdots \int_{\sigma_k > T} w_{pr}(\tau_1, \ldots, \sigma_k) \exp\left\{-\frac{1}{2N}\int_0^T (r - s)^2\, dt\right\} \times$$
$$\times\, d\tau_1 \ldots d\sigma_k. \tag{7}$$

Similarly, we find the a posteriori probability that an interval between pulses will exist at the instant of time $t = T$:

$$w_2(T) = K \sum_{k=1}^{M} \int_{\sigma_{k-1} < T} \cdots \int_{\tau_k > T} w_{pr}(\tau_1, \ldots, \tau_k) \exp\left\{-\frac{1}{2N}\int_0^T (r - s)^2\, dt\right\} \times$$
$$\times\, d\tau_1 \ldots, d\tau_k. \tag{8}$$

We shall now use more detailed a priori data on the statistical distribution of the transition points. If the number pairs (τ_i, σ_i), $i = 1, 2, \ldots$, form a Markov chain, then,

$$w_{pr}(\tau_1, \sigma_1, \ldots, \tau_k, \sigma_k) = w_{pr}(\tau_1, \sigma_1)\, w_{pr}(\tau_2, \sigma_2 \mid \tau_1, \sigma_1) \ldots$$
$$\ldots w_{pr}(\tau_k, \sigma_k \mid \tau_{k-1}, \sigma_{k-1}), \tag{9}$$

where $w_{pr}(\tau_1, \sigma_1)$ is the initial distribution, and $w_{pr}(\tau_i, \sigma_i \mid \tau_{i-1}, \sigma_{i-1})$ are the transition probabilities. If we assume that the durations of intervals and pulses are independent, the sequence of the transition points τ_1, σ_1, τ_2, $\sigma_2, \ldots$ will form a simple Markov chain, and the following expressions will hold:

$$w_{pr}(\tau_1, \ldots, \tau_k, \sigma_k) = w_{pr}(\tau_1)\, w_{pr}(\sigma_1 \mid \tau_1) \ldots w_{pr}(\tau_k \mid \sigma_{k-1})\, w_{pr}(\sigma_k \mid \tau_k),$$
$$w_{pr}(\tau_1, \ldots, \tau_k) = w_{pr}(\tau_1)\, w_{pr}(\sigma_1 \mid \tau_1) \ldots w_{pr}(\tau_k \mid \sigma_{k-1}), \tag{10}$$

where $w_{pr}(\tau_1)$ is the initial distribution, and $w_{pr}(\sigma_i \mid \tau_i)$ and $w_{pr}(\tau_i, \sigma_{i-1})$ are the transition probabilities.

In the equations

$$w_{pr}(\sigma_i \mid \tau_i) = p_1(\sigma_i - \tau_i), \quad w_{pr}(\tau_i \mid \sigma_{i-1}) = p_2(\tau_i - \sigma_{i-1}) \tag{11}$$

they are expressed in terms of the probability distribution density $p_1(T_{pu})$ for the pulse duration T_{pu} and a similar density $p_2(T_{in})$ for the intervals. In the case of steady-state signals, the functions $p_1(T_{pu})$ and $p_2(T_{in})$ are independent of $\underline{t}$; in the general case, such a dependence can be taken into account by considering the functions

$$w_{pr}(\sigma_i \mid \tau_i) = p_1(\sigma_i - \tau_i, \tau_i), \quad w_{pr}(\tau_i \mid \sigma_{i-1}) = p_2(\tau_i - \sigma_{i-1}, \sigma_{i-1})$$

Limiting our consideration to the case of a simple Markov chain, we shall substitute (10) in (7) and (8). If we denote

$$v_2(\tau_k) = \sum_{k=1}^{M} \int \ldots \int_{\sigma_{k-1} < \tau_k} w_{pr}(\tau_1)\, w_{pr}(\sigma_1 \mid \tau_1) \ldots w_{pr}(\tau_k \mid \sigma_{k-1}) \times$$
$$\times \exp\left\{-\frac{1}{2N}\int_0^{\tau_k} (\tau - s)^2\, dt\right\} d\tau_1 \ldots d\sigma_{k-1}, \tag{12}$$

$$v_1(\sigma_{k-1}) = \sum_{k=1}^{M} \int_{\tau_{k-1}<\sigma_{k-1}} \ldots \int w_{pr}(\tau_1)\, w_{pr}(\sigma_1 \mid \tau_2) \ldots w_{pr}(\sigma_{k-1} \mid \tau_{k-1}) \times$$
$$\times \exp\left\{-\frac{1}{2N}\int_0^{\sigma_{k-1}} (r-s)^2\, dt\right\} d\tau_1 \ldots d\tau_{k-1},$$

the results of the substitution will be given by

$$w_1(T) = K \int_{\tau_k<T} \int_{\sigma_k>T} v_2(\tau_k)\, w_{pr}(\sigma_k \mid \tau_k) \exp\left\{-\frac{1}{2N}\int_{\tau_k}^{T} (r-a)^2\, dt\right\} d\tau_k d\sigma_k.$$

$$w_2(T) = K \int_{\sigma_{k-1}<T} \int_{\tau_k>T} v_1(\sigma_{k-1})\, w_{pr}(\tau_k \mid \sigma_{k-1}) \exp\left\{-\frac{1}{2N}\int_{\sigma_{k-1}}^{T} (r+a)^2\, dt\right\} \times$$
$$\times\, d\sigma_{k-1} d\tau_k. \tag{13}$$

Equations (12) determine the $v_{1,2}$ functions in terms of the arguments $\tau = \tau_k$ and $\sigma = \sigma_{k-1}$. If we substitute τ and σ for the arguments of these functions, we readily see that they do not depend on k, since the summation was performed with respect to k. The first of the equations (12) can be written in the following form:

$$v_2(\tau) = \int_0^{\tau} d\sigma \sum_{k=1}^{M} \int_{\tau_{k-1}<\sigma} \ldots \int w_{pr}(\tau_1) \ldots w_{pr}(\sigma \mid \tau_{k-1})\, w_{pr}(\tau \mid \sigma) \times$$
$$\times \exp\left\{-\frac{1}{2N}\int_0^{\tau} (r-s)^2\, dt\right\} d\tau_1, \ldots, d\tau_{k-1}.$$

Hence, by using the second equation, we have:

$$v_2(\tau) = \int_0^{\tau} v_1(\sigma)\, w_{pr}(\tau \mid \sigma) \exp\left\{-\frac{1}{2N}\int_{\sigma}^{\tau} (r+a)^2\, dt\right\} d\sigma. \tag{14a}$$

Similarly, we obtain:

$$v_1(\sigma) = \int_0^{\sigma} v_2(\tau)\, w_{pr}(\sigma \mid \tau) \exp\left\{-\frac{1}{2N}\int_{\tau}^{\sigma} (r-a)^2\, dt\right\} d\tau. \tag{14b}$$

Thus, the summation in (12) is eliminated.

The latter equations serve for determining the functions v_1 and v_2, which can now be used for calculating w_1 and w_2. By introducing the density probabilities (11) and also the integral distribution functions

$$P_1(x) = \int_x^{\infty} p_1(t)\, dt, \qquad P_2(x) = \int_x^{\infty} p_2(t)\, dt, \tag{15}$$

the basic equations (13) and (14) can be written thus:

$$v_1(t) = \int_0^{t} p_1(t-\tau) \exp\left\{-\frac{1}{2N}\int_{\tau}^{t} (r-a)^2\, dt'\right\} v_2(\tau)\, d\tau,$$

$$v_2(t) = \int_0^{t} p_2(t-\tau) \exp\left\{-\frac{1}{2N}\int_{\tau}^{t} (r+a)^2\, dt'\right\} v_1(\tau)\, d\tau, \tag{16}$$

$$w_1(T) = K \int_0^T P_1(T-\tau) \exp\left\{-\frac{1}{2N}\int_\tau^T (r-a)^2\, dt'\right\} v_2(\tau)\, d\tau,$$

$$w_2(T) = K \int_0^T P_2(T-\tau) \exp\left\{-\frac{1}{2N}\int_\tau^T (r+a)^2\, dt'\right\} v_1(\tau)\, d\tau. \tag{17}$$

If we explicitly consider the initial instant of time t_0, the following initial conditions must be added to Eqs. (16):

$$v_1(\sigma) \to w_{pr}(\sigma), \quad v_2(\tau) \to w_{pr}(\tau) \quad \text{for} \quad \sigma, \tau \to t_0.$$

However, under steady-state conditions, the role of the initial conditions is not essential, and they can be left out of the consideration, assuming that the lower integration limit is $t_0 = -\infty$ in (16) and (17).

2. Block Diagram of the Optimal Receiver

The form of Eqs. (16) and (17) determines the block diagram of the receiver that is to perform the optimum nonlinear transformation of the received signal r(t) for the purpose of separating the useful signal. The $v_{1,2}(t)$ and $w_{1,2}(t)$ functions can be realized in the circuit as variable electrical signals. Instead of the $v_{1,2}(t)$ function, it is convenient to consider the functions

$$v_{1,2}(t) \exp\left\{\frac{1}{2N}\int_0^t (r^2+a^2)\, dt'\right\}$$

or the functions

$$V_{1,2}(t) = v_{1,2}(t) \exp\left\{\frac{1}{2N}\int_0^t (r+a)^2\, dt'\right\}. \tag{18}$$

If we introduce the notation

$$E(t) = 2\frac{a}{N}\int_0^t r(t')\, dt', \tag{19}$$

Eqs. (16) and (17) will assume the following form:

$$V_1(t) = e^{E(t)} \int_0^t p_1(t-\tau)\, e^{-E(\tau)} V_2(\tau)\, d\tau,$$

$$V_2(t) = \int_0^t p_2(t-\tau)\, V_1(\tau)\, d\tau,$$

$$w_1(t) = K' e^{E(t)} \int_0^t P_1(t-\tau)\, e^{-E(\tau)} V_2(\tau)\, d\tau, \tag{20}$$

$$w_2(t) = K' \int_0^t P_2(t-\tau)\, V_1(\tau)\, d\tau.$$

Figure 1 shows a block diagram which corresponds to the adduced equations. The device which produces a E(t) or $e^{\pm E(t)}$ signal comes first. Then there follows a system of units with feedback, which produces the $V_1(t)$ and $V_2(t)$ signals. This system consists of linear units with the transfer functions $p_1(t-\tau)$ and $p_2(t-\tau)$ and of units which perform multiplication by the functions $e^{E(t)}$ and $e^{-E(t)}$. In correspondence with Eqs. (20), the $V_1(t)$ and $V_2(t)$ signals are then transmitted to units with a similar structure, but without feedback, which correspond to the transfer functions

$P_1(t-\tau)$ and $P_2(t-\tau)$. If the signal value is estimated with respect to the maximum probability, the difference between the obtained functions must be fed to a threshold circuit (relay) P. The output signal $s_0(t) = a \operatorname{sgn}[w_1(t) - w_2(t)]$ actually represents the filtered-out useful signal. In this, the K' factor, which can always be eliminated by using the normalization condition $w_1(t) + w_2(t) = 1$, is of no importance. Instead of $V_1(t)$ and $V_2(t)$, we can also obtain functions that are proportional to these signals by choosing suitable amplification factors.

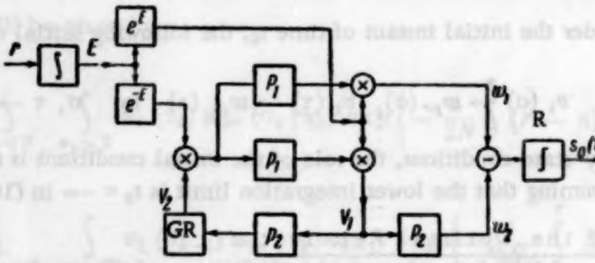

Fig. 1

The totality of the p_1 and p_2 units constitutes a closed-loop system with positive feedback. Self-oscillations, which are modulated by the $e^{\pm E}$ signal, will be excited in this system. Such conditions are normal for the operation of this system. In order to avoid an unlimited rise of the oscillations which occur in linear systems, it is advisable to provide a nonlinear element, the gain regulator GR, which limits the amplitude. It is controlled by a certain signal $|v_1|$ (or $|v_2|$), which is averaged over several periods.

It is understood that various receiver modifications can be provided for different actual cases in dependence on the technological conditions.

3. Filter Discrimination of Specially Shaped Telegraph Signals

Let us consider a telegraph signal with a somewhat more particular shape, for which the above results can be modified without affecting the basic principles.

Assume that the signal transitions from one state into another can occur only at the points $t_m = (m-1)T_0 + t_1$, where $m = 1, 2, \ldots$ is an integer, and T_0 is a fixed quantity (period) that is assigned in advance. The precise positions of the transition points are, however, not known, since the value of t_1 $(0 < t_1 \leq T_0)$, which coincides with the coordinate of the first of the indicated points, is not known.

The transition probabilities entering (10) are now different from zero only at the points t_m. In this case, although the above-presented theory can be strictly applied, it is, however, more convenient to use it in a somewhat different form.

Each of the periods is characterized by a constant value of the signal $s_m = s(t) = \pm a$, t_{m-1}. We shall assume that the quantities $s_1, s_2, \ldots$ represent a Markov chain with the a priori transition probabilities $w_{pr}(s_i \mid s_{i-1})$. Consequently the multidimensional a priori distribution of signal values during the first m periods will have the following form:

$$w_{pr}(s_1, \ldots, s_{m+1}) = w_{pr}(s_1)\, w_{pr}(s_2 \mid s_1) \ldots w_{pr}(s_{m+1} \mid s_m). \tag{21}$$

We shall assume that the value of t_1 is independent of $s_1, s_2, \ldots$ and that it has the a priori distribution density $w_{pr}(t_1)$.

We shall find the a posteriori distribution $w_{pr}(t_m, s_1, \ldots, s_{m+1})$ of the random quantities $t_m = t_1 + mT_0 - T_0$, $s_1, \ldots, s_{m+1}$ that corresponds to the observation in interval $0 < t < T$, while the end of the observation time and t_m pertain to the same period $(m-1)T_0 < T < mT_0$.

If, as before, the noise is assumed to be white noise, then, from (21) and by analogy with (1), we have:

$$w_{ps}(t_m, s_1, \ldots, s_{m+1})_T = Kw_{pr}(t_m - mT_0 + T_0)\, w_{pr}(s_1)\, w_{pr}(s_2 \mid s_1) \ldots$$

$$\ldots w_{pr}(s_{m+1} \mid s_m) \exp\left\{-\frac{1}{2N}\int_0^T (r - s)^2\, dt\right\}. \tag{22}$$

If we are interested in the signal value during the mth period or during the neighbouring periods, the summation in (22) must be performed with respect to the remaining signal values $s_1, s_2, \ldots$. It is convenient to introduce the function

$$v(t_m, s_m) = \sum_{s_1, \ldots, s_{m-1}} w_{pr}(t_m - mT_0 + T_0)\, w_{pr}(s_1)\, w_{pr}(s_2 \mid s_1) \ldots$$

$$\ldots w_{pr}(s_m \mid s_{m-1}) \exp\left\{-\frac{1}{2N}\int_0^{t_m} (r - s)^2\, dt\right\}. \tag{23}$$

By means of this function, the probabilities

$$w_{ps}(t_m, s_m)_T = \sum_{s_1, \ldots, s_{m-1}, s_{m+1}} w_{ps}(t_m, s_1, \ldots, s_{m+1}),$$

$$w_{ps}(t_m, s_{m+1})_T = \sum_{s_1, \ldots, s_m} w_{ps}(t_m, s_1, \ldots, s_{m+1}), \tag{24}$$

can be written as

$$w_{ps}(t_m, s_m) = K \sum_{s_{m-1}} w_{pr}(s_m \mid s_{m-1}) \exp\left\{-\frac{1}{2N}\int_{t_{m-1}}^{T} (r - s_m)^2\, dt\right\} \times$$

$$\times\, v(t_{m-1}, s_{m-1}) \quad \text{for} \quad t_m > T, \tag{25}$$

$$w_{ps}(t_m, s_{m+1}) = K \sum_{s_m} w_{pr}(s_{m+1} \mid s_m) \exp\left\{-\frac{1}{2N}\int_{t_m}^{T} (r - s_{m+1})^2\, dt\right\} v(t_m, s_m)$$

$$\text{for} \quad t_m < T. \tag{26}$$

In order to construct an optimum discriminating system, it is necessary to know the a posteriori probabilities that the signal will have a positive or negative value at the instant of time T. Let us denote these probabilities by w(T, a) and w(T, −a).

In view of the fact that the points t_m and T pertain to the same period $(m-1)T_0 < t < mT_0$, their sequence can have different orders. In correspondence with two different possibilities ($t_m > T$ and $t_m < T$), the above probabilities are composed of two components:

$$w(T, s) = \int_T^{mT_0} [w_{ps}(t_m, s_m)_T]_{s_m = s}\, dt_m + \int_{(m-1)T_0}^{T} [w_{ps}(t_m, s_{m+1})_T]_{s_{m+1} = s}\, dt_m. \tag{27}$$

By substituting (25) and (26) in this expression and by combining both terms, we find:

$$w(T, s) = K \sum_{s'} \int_{T-T_0}^{T} w_{pr}(s \mid s') \exp\left\{-\frac{1}{2N}\int_\tau^T (r - s)^2\, dt\right\} v(\tau, s')\, d\tau. \tag{28}$$

Thus, the probabilities in which we are interested are expressed by the function (23). Let us introduce the equation which serves for calculating the latter function. If we increase the number m by 1 in (23), we obtain:

$$v(t_{m+1}, s_{m+1}) = \sum_{s_1,\ldots,s_m} w_{pr}(t_{m+1} - mT_0)\, w_{pr}(s_1)\, w_{pr}(s_2 \mid s_1) \ldots$$
$$\ldots w_{pr}(s_{m+1} \mid s_m) \exp\left\{-\frac{1}{2N}\int_0^{t_m}(r-s)^2\,dt - \frac{1}{2N}\int_{t_m}^{t_{m+1}}(r - s_{m+1})^2\,dt\right\}. \tag{29}$$

By comparing (29) and (23), we arrive at the following relationship:

$$v(t_{m+1}, s_{m+1}) = \exp\left\{-\frac{1}{2N}\int_{t_{m+1}-T_0}^{t_{m+1}}(r - s_{m+1})^2\,dt\right\} \times$$
$$\times \sum_{s_m} w_{pr}(s_{m+1} \mid s_m)\, v(t_{m+1} - T_0, s_m), \tag{30}$$

which is the usual relationship in the theory of conditional Markov processes. This equation is similar to Eq. (14), and it serves for the step-by-step determination of the v(t, s) function.

Equations (28) and (30) indicate what optimum transformations of the input signal must be performed in order to obtain the a posteriori probabilities w(T, a) and w(T, −a) at the system's output, which are to be used for determining the filtered-out value $s_0(t)$ of the signal at the instant of time t = T.

It should be borne in mind that the a posteriori information will be more accurate and that the filter discrimination error will be smaller if we know the behavior of the signal r(t) for sometime after the moment $t = t^*$ for which the useful signal value is determined ($t^* < T$). In this case, the filtered-out signal will have a certain time lag $T - t^*$ with respect to the received signal, which is compensated by an improvement in the quality of filter discrimination. In the case under consideration, it is convenient to choose T_0 as the time lag. Let us derive the optimum equations for filter discrimination for this lag. We shall find the a posteriori signal probabilities at the instant of time which precedes (by one period) the end of observation. If we denote these probabilities by

$$w_{1,2}(T - T_0) = \mathrm{P}\{s(T - T_0) = \pm a \mid r(\tau),\ 0 \leqslant \tau \leqslant T\}, \tag{31}$$

we find by analogy with (27):

$$w_{1,2}(T - T_0) = \int_{T-T_0}^{mT_0-T_0} w_{ps}(t_{m-1}, s_{m-1})\,dt_{m-1} + \int_{mT_0-T_0}^{T} w_{ps}(t_m, s_m)_T\,dt_m =$$
$$= \int_{T}^{mT_0} w_{ps}(t_m, s_{m-1})_T\,dt_m + \int_{mT_0-T_0}^{T} w_{ps}(t_m, s_m)_T\,dt_m, \tag{32}$$

where $s_{m-1} = \pm a$, $s_m = \pm a$.

From (22), by summation with respect to all s with the exception of s_{m-1} and s_m, we readily find:

$$w_{ps}(t_m, s_m)_T = K v(t_m, s_m) \sum_{s_{m+1}} \exp\left\{-\frac{1}{2N}\int_{t_m}^{T}(r - s_{m+1})^2\,dt\right\} w_{pr}(s_{m+1} \mid s_m) \quad \text{for} \quad t_m < T,$$

$$w_{ps}(t_m, s_{m-1})_T = K v(t_m - T_0, s_{m-1}) \sum_{s_m} \exp\left\{-\frac{1}{2N}\int_{t_m-T_0}^{T}(r - s_m)^2\,dt\right\} \times w_{pr}(s_m \mid s_{m-1}) \quad \text{for} \quad t_m > T. \tag{33}$$

By substituting the latter expressions in (32) and by combining both terms, we find:

$$w_{1,2}(T - T_0) = K \int_{T-T_0}^{T} v(\tau, \pm a) \sum_s \exp\left\{-\frac{1}{2N}\int_{\tau}^{T}(r - s)^2\,dt\right\} w_{pr}(s \mid \pm a)\,d\tau \tag{34}$$

The obtained equations (30) and (34) can be used for solving the problem of optimum filter discrimination. We shall apply them to the important particular case where the telegraph messages during the neighbouring periods are a priori independent and equally probable. This means that

$$w_{pr}(\pm a \mid a) = \frac{1}{2}, \quad w_{pr}(\pm a \mid -a) = \frac{1}{2}, \quad w_{pr}(\pm a) = \frac{1}{2}. \tag{35}$$

For this, we obtain from (30) and (34):

$$v_{1,2}(t) \equiv v(t, \pm a) = \frac{1}{2}\exp\left\{-\frac{1}{2N}\int_{t-T_0}^{t}(r \mp a)^2\, dt'\right\}[v_1(t-T_0) + v_2(t-T_0)], \tag{36}$$

$$w_{1,2}(T-T_0) = \frac{1}{2}K\int_{T-T_0}^{T} v_{1,2}(\tau)\left[\exp\left\{-\frac{1}{2N}\int_{\tau}^{T}(r-a)^2\, dt\right\} + \exp\left\{-\frac{1}{2N}\int_{\tau}^{T}(\tau+a)^2\, dt\right\}\right]d\tau. \tag{37}$$

As before, we shall introduce the functions $V_{1,2}(t)$ and $E(t)$ by means of (18) and (19). Then, Eqs. (36) and (37) will assume the following form:

$$\begin{gathered} V_1(t) = \frac{1}{2}e^{E(t)}e^{-E(t-T_0)}[V_1(t-T_0) + V_2(t-T_0)], \\ V_2(t) = \frac{1}{2}[V_1(t-T_0) + V_2(t-T_0)], \\ w_{1,2}(T-T_0) = K'e^{E(t)}\int_{T-T_0}^{T} V^{-E(\tau)}V_{1,2}\,d\tau + K'\int_{T-T_0}^{T} V_{1,2}(\tau)\,d\tau. \end{gathered} \tag{38}$$

These relationships represent variants of Eqs. (20) in application to our case. They can be used for constructing the optimal receiver whose block diagram is shown in Fig. 2. Instead of linear units with the transfer functions $p_1(t-\tau)$ and $p_2(t-\tau)$, the diagram now contains lag units LU with the lag time T_0. Instead of units with the transfer functions $p_1(t-\tau)$ and $p_2(t-\tau)$, we now have units with the following transfer function:

$$G(t-\tau) = \begin{cases} G_0 & \text{for} \quad 0 < t-\tau < T_0, \\ 0 & \text{for} \quad t-\tau < 0 \text{ and } t-\tau > T_0. \end{cases} \tag{39}$$

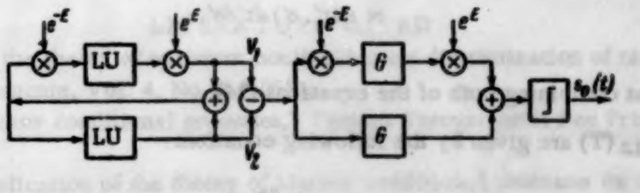

Fig. 2

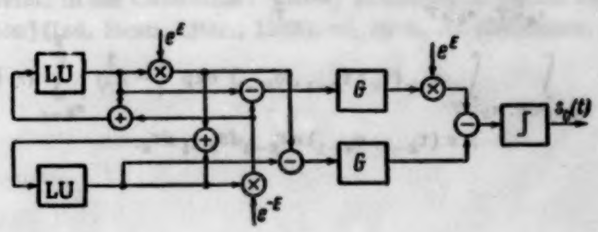

Fig. 3

Moreover, in comparison with the circuit of Fig. 1, the circuit of Fig. 2 is able to perform additional summation and subtraction operations. The circuit contains four units for multiplication by $\exp(\pm E)$; however, by modifying the circuit in the manner shown in Fig. 3, it is possible to reduce the number of such units to three for the same number of other operations. It is understood that other variants of the circuit can be obtained.

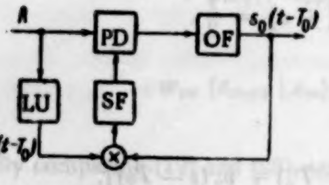

Fig. 4

As before, the signal which is proportional to the difference $w_1(t-T_0)-w_2(t-T_0)$ between the probabilities is fed to the relay which supplies the filtered-out signal $s_0(t-T_0)$ that has been delayed by one period.

In conclusion, we shall consider the case where the telegraph signal has a high-frequency filling, i.e., where it can be written in the form

$$S(t) = s(t)\cos(\omega_0 t + \varphi_0), \tag{40}$$

where s(t) is a signal of the type considered above. In a slightly more complicated form, the theory presented above can also be applied to this case. We shall indicate the direction to be followed in solving this problem in the case where the frequency ω_0, and the phase φ_0, are sufficiently stable.

The signal r(t), which coincides, or almost coincides, with s(t) + n(t), where n(t) is a certain new noise, is first separated from the received signal R(t) = S(t) + N(t) by means of phase detector PD (Fig. 4). This signal is supplied to the optimal filter OF, which is of the same type that has been described above. The result of filter discrimination $s_0(t-T_0)$, which is close to $s(t-T_0)$, is obtained at the output. The indicated filtered-out signal can be used for producing the reference voltage necessary for the phase detector's operation. This voltage is produced by multiplying $R(t-T_0)$ by $s_0(t-T_0)$ and by transmitting the product through a selective filter, SF (for instance, an oscillating circuit), which is tuned to the frequency ω_0.

Thus, the optimal filtering system that was calculated on the basis of the above theory represents a portion of a more complex system. The same situation can also be found in other cases.

APPENDIX

Generalization for the Case of Correlated Pulses and Correlated Noise

If the correlations existing between the durations of intervals and pulses must be taken into account, it is necessary to consider the sequence $\tau_1, \sigma_1, \sigma_2, \ldots$ as a complex Markov chain and to use, for instance, Eq. (9) instead of Eq. (10). The theory presented here can be extended to this case, but the corresponding expressions become more cumbersome. Thus, the function $v = v(\tau, \sigma)$ will now depend on two arguments, and it will satisfy the equation

$$v(\tau,\sigma)=\int_0^\tau\int_{\tau'}^\tau w_{pr}(\tau,\sigma\mid\tau',\sigma')\exp\left\{-\frac{1}{2N}\int_{\sigma'}^{\tau}(r+a)^2\,dt-\frac{1}{2N}\int_{\tau}^{\sigma}(r-a)^2dt\right\}\times$$

$$\times\, v(\tau',\sigma')\,d\tau'd\sigma',$$

which can be considered as combining both of the equations (14).

The probabilities $w_{1,2}(T)$ are given by the following equations:

$$w_1(T)=K\int\limits_{\tau_k<T,}\int\limits_{\sigma_k>T}\exp\left\{-\frac{1}{2N}\int_T^{\sigma_k}(r-a)^2dt\right\}v(\tau_k,\sigma_k)\,d\tau_k d\sigma_k,$$

$$w_2(T)=K\int\limits_{\sigma_{k-1}<T,}\int\limits_{\tau_k>T}w_{pr}(\tau_k\mid\tau_{k-1},\sigma_{k-1})\exp\left\{-\frac{1}{2N}\int_{\sigma_{k-1}}^{T}(r+a)^2dt\right\}\times$$

$$\times\, v(\tau_{k-1},\sigma_{k-1})\,d\tau_{k-1}d\sigma_{k-1}\,d\tau_k.$$

While the expressions

$$\text{const}\, w_{ps}\,(\tau|\sigma) = w_{pr}\,(\sigma|\tau)\exp\left\{-\frac{1}{2N}\int_{\sigma}^{\tau}(r+a)^2dt\right\},$$

$$\text{const}\, w_{ps}\,(\sigma|\tau) = w_{pr}\,(\tau|\sigma)\exp\left\{-\frac{1}{2N}\int_{\tau}^{\sigma}(r-a)^2dt\right\}$$

before played the role of a posteriori (un-normalized) transition probabilities, the following expressions will now represent such transition probabilities:

$$\text{const}\, w_{ps}\,(\tau,\sigma\,|\,\tau',\,\sigma') = w_{pr}\,(\tau,\sigma\,|\,\tau',\sigma')\exp\left\{-\frac{1}{2N}\int_{\sigma'}^{\tau}(r+a)^2dt -\right.$$

$$\left.-\frac{1}{2N}\int_{\tau}^{\sigma}(r-a)^2dt\right\}.$$

We shall touch upon the problem of extending this theory to the case of correlated noise. Generally speaking, the correlation inherent in noise results in the fact that the transition points do not form a Markov chain a posteriori. In this case, one must consider complex a posteriori chains. It is of interest to analyze the particular case where the noise correlation time considerably exceeds the sum $\tau_{cor} \ll T_{pu} + T_{in}$ of the average pulse and interval durations (however this time can be comparable to T_{pu} or T_{in}). In this case, the two-dimensional a priori Markov chain (9) remains the same a posteriori, and the a posteriori transition probability is given by

$$\text{const}\, w_{ps}\,(\tau,\sigma\,|\,\tau',\sigma') = w_{pr}(\tau,\sigma\,|\,\tau',\sigma')\exp\left\{-\frac{1}{2N}\int_{\sigma'}^{\tau}\int_{\sigma'}^{\tau}[r(t)+a]A(t',t)[r(t')+a]-dtdt'\right.$$

$$-\frac{1}{N}\int_{\sigma'}^{\tau}\int_{\tau}^{\sigma}[r(t)+a]\,A\,(t,t')\,[r(t')-a]\,dtdt' -$$

$$\left.-\frac{1}{2N}\int_{\tau}^{\sigma}\int_{\tau}^{\sigma}[r\,(t)-a]\,A\,(t,t')\,[r\,(t')-a]\,dtdt'\right\}.$$

Here, the kernel A(t, t') is the inverse of the noise autocorrelation function

$$\int A\,(t,t')\,\overline{n\,(t')\,n\,(t'')}\,dt' = \delta\,(t-t'').$$

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A STABILITY CRITERION BASED ON THE METHOD OF TWO HODOGRAPHS

W. Oppelt

(Darmstadt, Germany)

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A stability criterion for automatic control systems is considered, which is based on the separation of the hodograph (locus) of an open system into two separate hodographs. The criterion may be applied to continuous linear, nonlinear, and pulse control systems.

To test automatic control systems for stability, methods are used based on the analysis of the configurations of hodographs (of complex frequency characteristics) of closed or open systems. For open systems one examines the configuration of the hodographs of $F_0 = x_{out}/x_{in}$, where x_{out} and x_{in} denote the output and input magnitudes of the open system upon harmonic excitation of the input. Following Nyquist [1], one examines the position of the hodograph of F_0 relative to the "critical point" $P_c = 1$ (Fig. 1). In what follows we limit ourselves to the case of smooth hodographs, which occurs most frequently in practice. In this case the closed system is stable, if the hodograph of F_0 of the open system intersects the positive real axis in the interval between 0 and 1. To insure a sufficient amount of stability, it is required in most cases that the intersection point be between 0.1 to 0.4. If the hodograph F_0 passes through the point 1, then there arises an undamped vibration with frequency ω_c correspondingto the given point of the hodograph. In this case the system will be unstable at the boundary of the stability region.

The system is unstable if the hodograph intersects the real axis to the right of the point 1.

If the hodograph F_0 does not go as it should, then the elements of the control system should be changed so as to obtain a better positioning for the hodograph. The influence of the individual elements on the character of the curve is difficult to track down in general. This is easier to accomplish if one joins all constant elements of the control circuit into a link with frequency characteristic F_S and all the variable elements into a link with characteristic F_R.

Only a moderate number of elements of the system should be included in the link with characteristic F_R, since a simple hodograph should be formed for F_R for which it is easy to establish the connection between its configuration and the parameters of the variable elements.

The frequency characteristic of the open system F_0, on which the stability criterion of Nyquist is based, is determined by the expression $F_0 = -F_R F_S$. Therefore the stability of the closed system may be determined on the basis of the consideration of the hodographs of F_R and F_S. The hodograph of F_R may be obtained as the quotient x_{out}/x (Fig. 2).

For the input magnitude x_{in} we then find $x_{in}/x = -1/F_S$. The hodographs of F_R and $-1/F_S$ may be taken as measures for the input and output magnitudes respectively of the open system if one considers points on the two curves which correspond to the same frequency.

The passage of the Nyquist curve through the positive real axis corresponds to the intersection of the hodographs F_R and $-1/F_S$ at points with identical frequency, with a straight line issuing from the origin (dotted line in Fig. 2). In this case if the value of F_0 is less than the value $-1/F_S$, then the open system is stable. To obtain a sufficient amount of stability the ratio of the magnitudes $|F_R| / |-1/F_S|$ should, as before, lie between 0.1 and 0.4.

The criterion of Leonhard [2] (for linear systems) was given in this form. The limits of its applicability were investigated by Cremeter and Kolberg [3]. It is particularly significant that this method of two hodographs F_R and $-1/F_S$ may serve as basis for the investigation of nonlinear and pulse control systems.

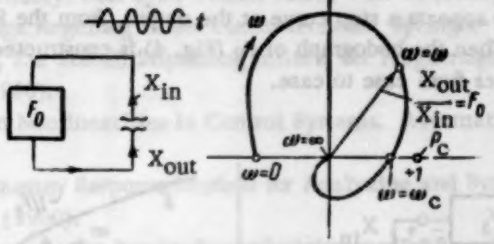

Fig. 1. Open linear control system and the hodograph of its frequency characteristic F_0.

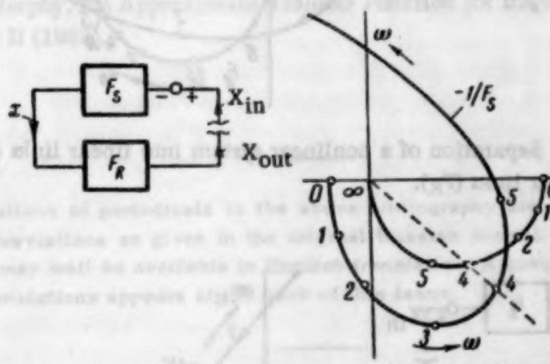

Fig. 2. Separation of a linear control system into mildly variable links (F_R), rigidly prescribed links (F_S) and elements changing sign. The corresponding separation of the hodograph of F_0 into two curves: F_R and $-1/F_S$.

Nonlinear Control Systems

We concentrate all linear elements into the link F_S, which corresponds to the hodograph $-1/F_S$. The link F_R is nonlinear; the hodograph of the equivalent impedance of the nonlinear element serves as hodograph of F_R in this case. The equivalent impedance is obtained from the first harmonic in the Fourier series expansion of the nonlinear characteristic of the variable output magnitude x_{out} when the input magnitude x varies harmonically. Thus this method is approximate.

The equivalent impedance in the general case depends on the amplitude x_0 and the frequency ω; for simple nonlinearities it depends only on x_0. Fig. 3 shows the case of ideally nonlinear link with friction. The intersection points of both hodographs correspond to damped vibrations in the open system. The frequency ω and amplitude x_0 of these undamped vibrations may be reckoned with reference to the marks on the hodographs. The stability of the vibration may be determined if one considers the motion of the system near the point of intersection of the hodographs. The point A corresponds to stable damped vibrations. Undamped vibrations may not be sustained at the point B, since a slight change in amplitude leads either to a stable vibration at a higher frequency (if the amplitude is increased), or to damping of the vibration (if it is decreased).

If the equivalent impedance associated with a complicated nonlinearity depends not only on the amplitude x_0, but also on the angular frequency ω, then our considerations are not changed.

The method of investigation of the stability of nonlinear control systems with the aid of equivalent impedance has been developed in many papers; above all in those of L. S. Gol'dfarb [4] and Kochenburger [5].

Pulse Control Systems

The method of two hodographs may also be applied in the construction of stability criteria for pulse control systems, if the pulse element is a fixed element. In Fig. 4 such a system is represented. The linear elements are also

grouped into a link which corresponds to the hodograph $-1/F_S$. The characteristics of the union of a pulse element with a delay element may be represented with the help of an equivalent impedance [6]. Upon harmonic excitation at the input of these elements there appears a step curve at the output, from the Fourier expansion of which one separates out the first harmonic. Then the hodograph of F_R (Fig. 4) is constructed. It underlies all pulse control systems, but the curve $-1/F_S$ changes from case to case.

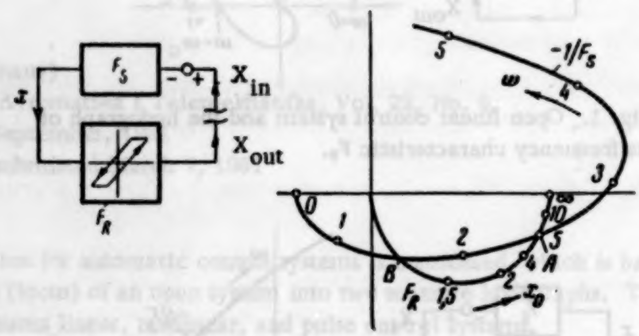

Fig. 3. Separation of a nonlinear system into linear links (F_S) and nonlinear links (F_R).

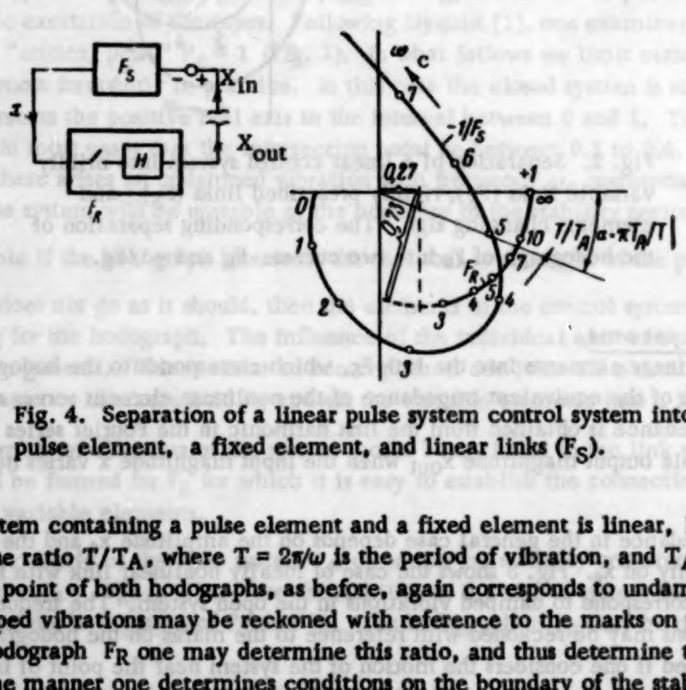

Fig. 4. Separation of a linear pulse system control system into a pulse element, a fixed element, and linear links (F_S).

Since a control system containing a pulse element and a fixed element is linear, F_R depends not only on the amplitude, but also on the ratio T/T_A, where $T = 2\pi/\omega$ is the period of vibration, and T_A is the interval between pulses. The intersection point of both hodographs, as before, again corresponds to undamped vibrations. The frequency ω_C of the undamped vibrations may be reckoned with reference to the marks on the curve $-1/F_R$. By the marks of T/T_A on the hodograph F_R one may determine this ratio, and thus determine the critical interval between pulses. In the same manner one determines conditions on the boundary of the stable region, since in linear systems the presence of undamped vibrations indicates the boundary of vibratory instability. Depending on the mutual position of the curves, a decrease or an increase in the interval between pulses will lead to a stable state. Figure 4 depicts the rare case when an instability arises for small T_A.

For moderate magnitudes of T/T_A the method becomes unreliable since then the first harmonic in the Fourier series yields a very rough approximation. However for ratios greater than $T/T_A = 3$, one nevertheless obtains admissible relations (see [7]). The method may be formulated directly for hodographs of F_0; i.e., for the Nyquist stability criterion; in this, F_0 is constructed according to F_R and F_S. However in this case the method loses clarity. It was given in this form in the papers of Brown and Murphy [8].

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All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. *Some or all of this periodical literature may well be available in English translation.* A complete list of the cover-to-cover English translations appears at the back of this issue.

ON THE STATISTICAL AUTONOMY OF DYNAMIC PROCESSES IN OPTIMALIZATION OBJECTS CONTAINING CONTROL SYSTEMS

R. I. Stakhovskii

(Moscow)

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It is demonstrated that for a broad class of n-dimensional optimalization objects which contain control systems and are controlled by the adjustment of controllers there exist conditions for which the systems that perform optimalization with respect to each individual input are statistically autonomous. These conditions can be found by automatic search for the extremum of the average value of the performance criterion if we study the magnitude of the correlation shifts as independent variables.

Introduction

Many optimalization objects are technological objects in which the most important parameters X_i $(i = 1,\ldots,n)$ (temperature, pressure, etc.) are stabilized by means of control systems. This permits the average values of the stabilized parameters to be maintained equal to the specified values irrespective of the appearance of random noise. Usually the specified values X_{i0} are determined on the basis of engineering considerations in such a way that a certain performance criterion (the efficiency, the productivity, the cost, etc.) reaches an extremal value. However, since with the passage of time the optimalization-object characteristics vary in unforseen fashion, it follows that the specified mode of operation for the object ceases to be optimal. The problem arises of performing a continuous automatic selection of X_{i0} in such a way that the performance criterion Q has an optimal value [1].

The realization of optimalization requires the measurement of the quantities which characterize the interrelationship between X_{i0} and Q. Usually in order to determine the characteristics of this interrelationship (i.e., the quantities dQ/dX_{i0}) the optimalization object is subjected to small trial perturbations at the inputs X_{i0}, and the response of the object output (i.e., the variation of Q [1]) is studied. Such a method is possible but not alwaysconvenient because of engineering considerations and because of the effects produced by the random noise which gives rise to random fluctuations of X_i and Q and thus masks the effects of the trial perturbations.

Therefore it is of importance to seek a method for determining the interrelationship between the random processes without requiring the application of any trial input. Here we make use of the statistical characteristics of the processes expressed in the form of mutual correlation functions [2] in order to determine the characteristics of the object which is subjected to random noise. This method permits us to investigate the interrelationships without disrupting the normal operation of the object. A knowledge of the object characteristics makes it possible to control them using the natural fluctuations of the system [3] as the trial movements.

However, in the case of multiple-loop systems the result of the correlation will contain more than just useful information on the interrelationship between the processes. Besides the useful component, it will contain an unknown component which depends on the effect of extraneous random factors if the latter are correlated with the processes with respect to which the mutual correlation function is determined. For example, it would be possible to study the fluctuations of the stabilized parameters X_i (i.e., the dynamic errors $x_i(t)$ of the controllers) by treating them as random trial steps which cause random fluctuations q(t) of the performance criterion [4]. Then the mutual correlation characteristics for the quantities $x_i(t)$ and the fluctuations of the performance criterion $q(t) - R_{X_i q}(\tau)$ could be used to determine a quantity analogous to $\partial Q / \partial X_{i0}$ which can easily be used as the control signal during the search process [1]. However, in the case of multiple-loop systems $x_i(t)$ are correlated with each other. Moreover, the causes for the appearance of q(t) in the general case are not just the quantities $x_i(t)$ but the noise signals proper which are correlated with $x_i(t)$.

In this regard it is useful to study the conditions for which the quantity $R_{X_{iq}}(\tau)$ would be proportional to $\partial Q_0 / \partial X_{i0}$ (Q_0 is the average value of the performance criterion); i.e., we wish to study the conditions for which this quantity would change sign for the transition of each individual function $Q_0(X_{i0})$ through the extremum. Under these conditions other input signals are treatedas aparticular case of noise. This permits us to "decouple" the optimalization system so that we can treat individual inputs. Here the indicated conditions are the autonomy conditions (in the statistical sense) for systems which perform optimalization with respect to the individual inputs.

Thus we are treating a particular case of the optimalization problem; the special feature of this problem is the presence of noise in an object which is controlled by means of setting the controllers.

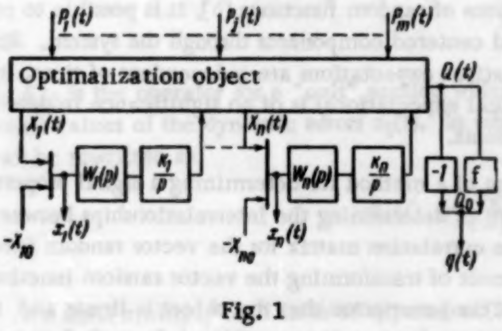

Fig. 1

Statement of the Problem

Figure 1 shows an optimalization object which is controlled by n control systems. This means that the object contains n technological parameters, each of which is stabilized near a specified value X_{i0}. Each control system consists of a pick-up device for a stabilized parameter, which produces a quantity representing the stabilized parameter $X_i(t)$ at its output. After comparing $X_i(t)$ with the standard value the system produces a quantity $x_i(t)$ which is amplified byablock with the transfer function $W_i(p)$ and is applied to an integrator (an actuator) with the transfer function k_i/p. The integrator output is a certain point in the object at which the control signal is applied. The section of the control loop which closes the system consists of that sector of the object being optimalized whose boundaries are defined by the points at which the control input is applied and the stabilized parameter is measured. The transfer function must be such that the control system is stable. All n control systems are mutually coupled; i.e.,

$$x_i = x_i\,(t, x_1, \ldots, x_n).$$

The optimalization object is subjected to m random noise signals $P_j(t)$. The noise signals are assumed to be stationary random processes. We introduce a certain performance criterion Q which in general case is also a random process. The magnitude of the criterion can be measured directly in the object, or it can be determined continuously by a high speed computer on the bases of measurements of the instantaneous values of definite output coordinates of the objects. In particular, these may be the stabilized parameters $X_i(t)$. If a computer is used, then it is included in the concept of the optimalization object. As far as the average value of the performance criterion Q_0 (its mathematical expectation) is concerned, we make the assumption that it is a nonlinear function of the adjustments $X_{10}, \ldots, X_{n0}$ of the controllers which has one extremum. The value of Q_0 is extracted from Q(t) by means of an averaging filter W_f. The fluctuations $q(t) = Q(t) - Q_0$ of the performance criterion are obtained by summing Q_0 and the inverted value of Q(t).

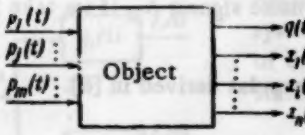

Fig. 2

Due to the effect of the random noise signals $P_j(t)$ which are applied to the object, the object is in a state for which continuous random fluctuations occur about a certain average mode which is defined by the adjustments X_{i0} of the controllers. Here $P_j(t)$ can be treated as input perturbations, and $X_i(t)$ (or the dynamic errors $x_i(t)$ and Q(t)) can be treated as output random processes. Thus we can write

$$\begin{aligned} X_i\,(t) &= X_{i0} + x_i\,(t), \\ Q\,(t) &= Q_0 + q\,(t), \\ P_j\,(t) &= P_{j0} + p_j\,(t), \end{aligned} \tag{1}$$

where X_{i0} is the value for the setting of the ith controller, Q_0 is the mathematical expectation of the performance criterion, P_{j0} is the mathematical expectation of the jth random noise signal, $x_i(t)$ is the dynamic error of the ith

controller which is a centered random process, q(t) are the fluctuations of the performance criterion relative to the median level Q_0, and $p_j(t)$ are the fluctuations of the jth random noise signal relative to P_{j0}.

The problem consists of finding the extremum point for the function $Q_0(X_{10}, \ldots, X_{n0})$ using only the values $x_i(t)$ and q(t).

We shall assume that $p_j(t)$, $x_i(t)$, q(t) are small, and that X_{i0} (before search is begun) are independent of time. These propositions make it possible to study an essentially nonlinear optimalization object as an object which is "linear in the small" if the set X_{i0} does not correspond to the extremum point of the function $Q_0(X_{10}, \ldots, X_{n0})$. In that case, in accordance with the theory of linear transformations of random functions [5], it is possible to perform a separate study of the passage of mathematical expectations and centered components through the system. Since according to the definition accepted for the time being mathematical expectations are independent of time, it follows that the nonlinearity of the object "in the large" (for mathematical expectations) is of no significance in determining the mutual correlation functions for the centered components.

One of the purposes of the present analysis is the realization of a method for determining a signal proportional to the partial derivative $\partial Q_0 / \partial X_{i0}$. We shall study the problem of determining the interrelationships between $x_i(t)$ and q(t) as the problem of determining certain components of the correlation matrix for the vector random function $\mathbf{X}_Q(t)$ whose components are $x_i(t)$ and q(t). Here $\mathbf{X}_Q(t)$ is the result of transforming the vector random function $\mathbf{P}(t)$ whose components are $p_j(t)$ (Fig. 2). This problem was solved on the assumption that the object is linear and is characterized by the matrix of "transfer" operators A_{ji} which consists of m(n + 1) terms [5]. A "transfer" operator A_{ji} is defined as an operator which represents the effect of the function $p_j(t)$, which is treated as an input perturbation, on the ith component of the function $X_Q(t)$ (i = 1, . . . , n + 1).

The difference between this treatment and the analysis performed in [5] resides in the fact that in addition to the matrix of "transfer" operators A_{ji} the object has a matrix of the operators A_{il} (i, l = 1, . . . , n + 1). The operators A_{il} are defined as operators which represent the effect of the input i on the output l. The operators A_{il} can be subdivided into two groups. First, there exist n "useful" operators A_{iq} which depend on the unknown quantities $\partial Q_0 / \partial X_{i0}$. Second, there exist operators which reflect the interrelationship between the dynamic errors of the control systems. Moreover, if the noise signals $p_j(t)$ are correlated, then there also exist the operators A_{jr} (j, r = = 1, . . . , m) [6].

In fact, if there are two correlated processes, for example $p_j(t)$ and $p_r(t)$, then in each of them it is possible to isolate the part which is not correlated with the other process. Here the correlated part is obtained using a certain equivalent operator which connects the measurement points $p_j(t)$ and $p_r(t)$. This means that without violating the general nature of our analysis we can study the case of statistically independent noise signals A_{jr} if we take into account the presence of the operators A_{jr}.

All of the concepts cited above do not make it possible for us to use the formulas derived in [5].

On the Existence of the Autonomy Conditions

We shall study the case when the only input is the noise $p_j(t)$ and we shall be concerned with the properties of the mutual correlation function $R_{x_iq}(\tau)$. Here (making use of the linearization method [5]) we shall assume that the system is linear for small values of the fluctuations $p_j(t)$, $x_i(t)$, q(t). The assumption that these quantities are small is justified by the very existence of control systems which reduce the fluctuations of the stabilized parameters to a minimum.

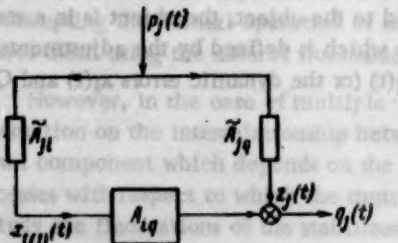

Fig. 3

We shall transform the network in Fig. 2 (treating only the jth noise signal alone) with respect to the ith "useful" operator (Fig. 3). It should be noted that the fluctuations q(t) consist of two components: one of them is caused directly by the dynamic error $x_i(t)$ which is transformed by the operator A_{iq}, and the other is caused by the noise $p_j(t)$ transformed by the generalized operator $\tilde{A}_{jq}$.

Thus we can write

$$q_j(t) = \{A_{iq} x_{i(j)}(t)\} + z_j(t), \tag{2}$$

where $z_j(t)$ is that component of the oscillations of the performance criterion which is caused by the noise $p_j(t)$ which is transformed by the operator A_{jq}; $x_{i(j)}(t)$ is that component of the dynamic error of the ith controller which is produced by the jth noise signal; $q_j(t)$ is that component of the performance criterion fluctuations which is caused by the jth noise signal.

The transformation shown in Fig. 3 can always be performed for any form of the linear operators A_{jr}, A_{ji}, A_{il}. The operator A_{iq}, which is a particular case of the operator A_{il} $(l = q)$, can be written as

$$A_{iq} = \frac{\partial Q_0}{\partial X_{i0}} A^1_{iq},$$

where A^1_{iq} is the operator for a "unit" section which has the same dynamic properties as the sector $i-q$ and is linear for small values of the dynamic errors $x_i(t)$. In other words, A^1_{iq} has a static gain equal to unity. Then expression (2) can be rewritten as

$$q_j(t) = \frac{\partial Q_0}{\partial X_{i0}} \{A^1_{iq}\, x_{i(j)}(t)\} + z_j(t). \tag{3}$$

We shall multiply both sides of expression (3) by $x_{i(j)}(t + \tau)$ and shall find the mathematical expectation of both parts

$$M\,[x_{i(j)}(t+\tau)\, q_j(t)] = R_{x_i q_j}(\tau) = \frac{\partial Q_0}{\partial X_{i0}} M\,[x_{i(j)}(t+\tau)\,\{A^1_{iq} x_{i(j)}(t)\}] +$$
$$+ M\,[x_{i(j)}(t+\tau)\, z_j(t)] = \frac{\partial Q_0}{\partial X_{i0}} \overline{R^1_{x_i q_j}}(\tau) + \overset{*}{R}_{x_i q_j}(\tau),$$

where $\frac{\partial Q_0}{\partial X_{i0}} \overline{R^1_{x_i q_j}}(\tau)$ is the useful component of the function $R_{x_i q_j}(\tau)$ and is proportional to $\partial Q_0 / \partial X_{i0}$; $\overset{*}{R}_{x_i q_j}(\tau)$ is the interfering component of the function $R_{x_i q_j}(\tau)$ which must be determined by setting $A_{iq} = 0$.

Before we proceed to a further analysis we shall determine the properties of the generalized operators $\tilde{A}_{ji}$. We shall juxtapose each operator with a pulse weighting function $\tilde{g}_{ij}(t)$. It can be assumed that they characterize the segments of the control system between the point at which the arbitrary noise is applied and the point at which the stabilized parameter is measured. If the control system is astatic and stable, then $\tilde{h}_{ji}(0) = 0$ and $\tilde{h}_{ji}(\infty) = 0$, where $\tilde{h}_{ji}(t)$ is the response of the above-mentioned segment to a unit step perturbation. If the control system is static and its open-loop system has a high gain, then $\tilde{h}_{ji}(\infty) \approx 0$. Then we can write

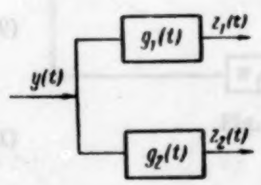

Fig. 4

$$\tilde{h}_{ji}(\infty) = \int_0^\infty g_{ji}(t)\, dt = 0. \tag{4}$$

This means that the graphs of the pulse weighting functions for all of the above-mentioned segments of the object intersect the time axis at least once.

We shall now study the mutual correlation function between the results of transforming a certain random process $y(t)$ using two sections (Fig. 4), one of which has a weighting function $g_2(t)$ with the property (4). In that case we can write

$$z_2(t) = \int_0^\infty y(t-\theta)\, g_2(\theta)\, d\theta. \tag{5}$$

Multiplying both sides of expression (5) by $z_1(t + \tau)$ and finding the mathematical expectations, we obtain

$$M\,[z_1(t+\tau)\, z_2(t)] = R_{z_1 z_2}(\tau) = M\,[z_1(t+\tau) \int_0^\infty y(t-\theta)\, g_2(\theta)\, d\theta].$$

Changing the sequence of the operations of integration and mathematical expectation, we find

$$R_{z_1z_2}(\tau) = \int_0^\infty g_2(\theta)\,d\theta M\,[z_1(t+\tau)\,y(t-\theta)] = \int_0^\infty R_{z_1y}(\tau-\theta)\,g_2(\theta)\,d\theta.$$

We shall now take into account the fact that $g_2(t)$ has the property (4) and shall find the magnitude of the integral of the function $R_{z_1z_2}(\tau)$:

$$\int_{-\infty}^{\infty} R_{z_1z_2}(\tau)\,d\tau = \int_{-\infty}^{\infty} d\tau \int_0^\infty R_{z_1y}(\tau-\theta)\,g_2(\theta)\,d\theta. \tag{6}$$

We shall transform expression (6); for this purpose we alter the sequence of integration

$$\int_{-\infty}^{\infty} R_{z_1z_2}(\tau)\,d\tau = \int_0^\infty g_2(\theta)\,d\theta \int_{-\infty}^{\infty} R_{z_1y}(\tau-\theta)\,d\tau. \tag{7}$$

The inner integral in the right side of expression (7) is equal to a constant, since it represents an area bounded by the graph for the mutual correlation function $R_{z_1y}(\tau)$ shifted along the τ axis by an amount equal to the variable parameter θ. The magnitude of the area is equal to a finite quantity, since $R_{z_1y}(-\infty) = R_{z_1y}(\infty) = 0$ and $R_{z_1y}(\tau)$ is a bounded function. In this regard the inner integral in the right side of expression (7) can be treated as a constant multiplier c. Thus,

$$\int_{-\infty}^{\infty} R_{z_1z_2}(\tau)\,d\tau = c\int_0^\infty g_2(\theta)\,d\theta = 0. \tag{8}$$

Assuming now that $y(t) = p_j(t)$, $z_1(t) = z_j(t)$ and $z_2(t) = x_{i(j)}(t)$, we find that $R^{*}_{x_iq_j}(\tau)$ has the property (8). This means that there exists at least one finite value τ_{i0} for which $R^{*}_{x_iq_j}(\tau_{i0}) = 0$. The resulting derivation is valid for any noise $p_j(t)$. Making the transition to the case where m noise signals are involved, we note that both $x_i(t)$ and $q(t)$ can be written in the following form in accordance with the superposition principle:

$$x_i(t) = \sum_{j=1}^{m} x_{i(j)}(t), \tag{9}$$

$$q(t) = \sum_{j=1}^{m} q_j(t) = \sum_{j=1}^{m}\left[\frac{\partial Q_0}{\partial X_{i0}}\{A^1_{iq}x_{i(j)}(t)\} + z_j(t)\right]. \tag{10}$$

Thus in determining the function $R_{x_iq}(\tau)$ [for this purpose it is necessary to multiply the right sides of expressions (9) and (10) and then average the results] we find that $2m^2$ components appear; each of these will represent the result of correlation between any pair $x_{i(j)}(t)$ and $q_l(t)$. It is easy to demonstrate that of the $2m^2$ components of the function $R_{x_iq}(\tau)$ only 2m will be nonzero. In fact, the results of the transformations of statistically independent processes are themselves statistically independent. Therefore we can consider just the components $(\partial Q_0/\partial X_{i0})R^1_{x_iq_j}(\tau)$ and $R^{*}_{x_iq_j}(\tau)$. Thus,

$$R_{x_iq}(\tau) = \sum_{j=1}^{m}\frac{\partial Q_0}{\partial X_{i0}}\overline{R}^1_{x_iq_j}(\tau) + \sum_{j=1}^{m}R^{*}_{x_iq_j}(\tau) = \frac{\partial Q_0}{\partial X_{i0}}\overline{R}^1_{x_iq}(\tau) + R^{*}_{x_iq}(\tau), \tag{11}$$

where

$$\overline{R}^1_{x_iq}(\tau) = \sum_{j=1}^{m}\overline{R}^1_{x_iq_j}(\tau),\quad R^{*}_{x_iq}(\tau) = \sum_{j=1}^{m}R^{*}_{x_iq_j}(\tau).$$

We can demonstrate that the mutual correlation function $R^*_{x_i q}(\tau)$ has the property (8). In fact,

$$\int_{-\infty}^{\infty} R^*_{x_i q}(\tau)\,d\tau = \int_{-\infty}^{\infty} \sum_{j=1}^{m} R^*_{x_i q_j}(\tau)\,d\tau = \sum_{j=1}^{m} \int_{-\infty}^{\infty} R^*_{x_i q_j}(\tau)\,d\tau = 0.$$

This property of the function $R^*_{x_i q}(\tau)$ makes it possible for us to state that for each function $R_{x_i q}(\tau)$ there exists at least one value τ_{i0} such that

$$R_{x_i q}(\tau_{i0}) = \frac{\partial Q_n}{\partial X_{i0}} \overline{R^1_{x_i q}}(\tau_{i0}). \tag{12}$$

This allows us to use $R_{x_i q}(\tau_{i0})$ as a measure of the magnitude of the sought for partial derivative.

Determining the Conditions for Statistical Autonomy by the Method of Automatic Search

We shall demonstrate that the quantities τ_{i0}, for which automatic systems that achieve optimalization with respect to separate inputs are autonomous in the statistical sense, can be found by means of automatic search for the minimum magnitude of the steady-state value of the performance criterion $Q_{0st.st.}$ treated as a function of the correlation shifts τ_i. Under these conditions the entire automatic optimalization system is of the form shown in Fig. 5. The entire system consists of n separate systems which perform optimalization with respect to each separate input of the system shown in Fig. 1; here the input is defined as the setting of the corresponding controller. Each individual system consists of a block that achieves controlled delay, a multiplier, a low-pass filter, and an integrator [7]. The average value Q_0 is measured at the output of the low-pass averaging filter.

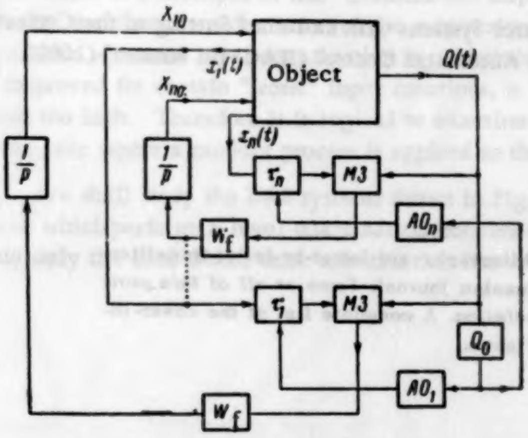

Fig. 5

If we close each separate automatic optimalization system (for certain arbitrary values of τ_i) through its actuator which consists of an integrator with a time constant appreciably greater than the time constant of the low-pass averaging filter, then the stable state of each individual system (for the condition that negative feedback is used) is obtained if the input voltage of each integrator is equal to zero on the average. This will correspond to the case where the left side of expression (11) is equal to zero. Therefore a condition for the stable state of each individual automatic optimalization system will be

$$\left(\frac{\partial Q_0}{\partial X_{i0}}\right)_{X_{i0}=X_{i0\,\mathrm{st.st.}}} = -\frac{R^*_{x_i q}(\tau)}{\overline{R^1_{x_i q}}(\tau)} \qquad (i = 1, \ldots, n). \tag{13}$$

The system of equations (13) will define a steady-state point in the space of the independent parameters X_{i0}. This steady-state point will correspond to the specific value $Q_{0st.st.}$; i.e., it will correspond to the average value of the performance criterion in a steady-state mode. It is natural that when τ_i varies the quantity $Q_{0st.st.}$ will also vary, since the steady-state point in the space of the parameters X_{i0} varies. Thus it is possible to study the quantity $Q_{0st.st.}$ as a function of the correlation shifts τ_i in the separate automatic optimalization system. If we assume for simplicity that the function $Q_{0st.st.}(\tau_1, \ldots, \tau_n)$ has one extremum point, then this extremum will coincide with the unknown extremum point for the function $Q_0(X_{10}, \ldots, X_{n0})$; here the values τ_i will coincide with τ_{i0}. In fact, for $\tau_i = \tau_{i0}$, $R^*_{x_i q}(\tau_{i0}) = 0$; therefore, $(\partial Q_0/\partial X_{i0})_{X_{i0} = X_{i0}\mathrm{st.st.}} = 0$. In order to find the extremal point we use a single-channel automatic optimalizer AO. The case where the function $Q_{0st.st.}(\tau_1, \ldots, \tau_n)$ has several extremums can be reduced to the case of a single extremum by an appropriate choice of the initial correlation shift. The problem of choosing the initial shifts is not studied in this paper.

SUMMARY

1. For a wide class of n-dimensional objects which are optimalized by changing the settings of controllers and which are subjected to random noise signals, there exist values of the correlation shifts in the mutual correlation functions for the dynamic errors of the controllers and the fluctuations of the performance criterion for which the systems performing automatic optimalization with respect to separate inputs are statistically autonomous. Thus an m-dimensional optimalization system reduces to m unidimensional systems.

2. These values of the correlation shifts can be found by means of automatic search for the extremum of the average value of the performance criterion if we treat the magnitudes of the correlation shifts as independent variables.

3. When the search method cited above is used (this method does not require the application of trial inputs to the objects) the extremum for the function $Q_0(X_{10}, \ldots, X_{n0})$ is reached automatically.

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All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. ***Some or all of this periodical literature may well be available in English translation.*** **A complete list of the cover-to-cover English translations appears at the back of this issue.**

ESTIMATING THE EFFECT OF LEVEL QUANTIZATION ON PROCESSES IN DIGITAL AUTOMATIC SYSTEMS WHEN A RANDOM INPUT SIGNAL IS USED

V. A. Volkonskii
(Moscow)

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The paper determines the probability characteristics for the level quantization noise in digital automatic systems when a Gaussian random process is applied to the system input and the quantization step is small compared to its mean square deviation.

In digital automatic systems (DAS) quantities are subjected to quantization with respect to level and with respect to time. Level quantization consists of rounding off the quantity (i.e., it consists of substituting the closest value which is a multiple of the "quantization step" Δ for it). In [1] we provide an estimate of the "quantization noise" – the maximum deviation of the output signal of a DAS from the output signal of the same system under conditions where no level quantization is performed (the limiting system for $\Delta \to 0$). Although this estimate cannot be improved for certain "worst" input functions, it is nevertheless true that in the majority of cases it proves to be much too high. Therefore it is logical to examine how the probability characteristics of the quantization noise vary in the case where a random process is applied to the input and the quantization step Δ tends toward zero.

We shall study the DAS systems shown in Figs. 1 and 2 where K and L denote linear elements, and AD denotes a unit which performs a level quantization (conversion of an analog quantity to a digital quantity). Here we shall study only the case where time quantization is not performed or when its effect can be neglected.

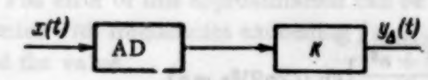

Fig. 1

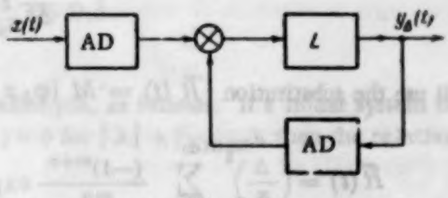

Fig. 2

We shall use x(t) and $y_\Delta(t)$ to denote the input and output signals. We shall assume that x(t) is a Gaussian stationary random process with zero mathematical expectation, a disperion σ_x^2 and a normalized correlation function r_t. We shall find the approximate form of the correlation function for the process:

$$\delta_\Delta (t) = y_\Delta (t) - y_0 (t),$$

where y_0 (t) is the output signal from the limiting system (the system without quantization).

In addition we shall introduce the following substitutions* :

* $x'(t)$ and $y_0'(t)$ denote the first derivatives of the processes x(t) and $y_0(t)$; r_0'' denotes the second derivative of the correlation function r_t at the point t = 0.

$$\sigma_y^2 = Dy_0(t),\ \beta = \frac{\sigma_x}{\Delta},\ \sigma_x'^2 = Dx'(t) = -r_0''\sigma_x^2,\ \sigma_{y_t}'^2 = Dy_0'(t),\ f_x = \frac{\sigma_x'}{\sigma_x},\ f_y = \frac{\sigma_y'}{\sigma_y}.$$

The quantities fx and fy are the mean square frequencies of the processes x(t) and $y_0(t)$. If $\varphi_x(\omega)$ is the spectral density of the process x(t), then

$$f_x = \sqrt{\int_{-\infty}^{\infty} \omega^2 \frac{\varphi_x(\omega)}{\sigma_x^2}\, d\omega}.$$

Here $\tilde{x}_\Delta$ shall denote the result of applying level quantization to the signal $\underline{x}$; i.e., it is the quantity closest to $\underline{x}$ which is a multiple of the quantization step:

$$\tilde{x}_\Delta = \left[\frac{x}{\Delta} + \frac{1}{2}\right]\Delta,$$

where [x] denotes the integer part of the number $\underline{x}$ and $\psi_\Delta x$ denotes the difference

$$\psi_\Delta x = x - \tilde{x}_\Delta. \tag{1}$$

1. The Case of an Open-Loop System

Assume that for large β the distribution of the quantity $\frac{\psi_\Delta x(t)}{\Delta}$ is close to a uniform distribution over the segments $[-1/2, 1/2]$. Therefore for $\beta \to \infty$,

$$\frac{1}{\Delta^2} D\psi_\Delta x(t) \to \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2\,dx = \frac{1}{12}$$

and

$$\frac{12}{\Delta^2} D\psi_\Delta x(t) \to 1.$$

We shall use the substitution $\overline{R}(t) = M[\psi_\Delta x(0), \psi_\Delta x(t)]$. In the Appendix we prove [cf. Eq. (13a)] that

$$\overline{R}(t) = \left(\frac{\Delta}{\pi}\right)^2 \sum_{m,\,n=1}^{\infty} \frac{(-1)^{m+n}}{mn} \exp\left[-(2\pi\beta)^2 \frac{m^2+n^2}{2}\right] \operatorname{sh}\left[(2\pi\beta)^2 r_t mn\right]. \tag{2}$$

We shall now study the normalized correlation function $\overline{r}(t) = \frac{12}{\Delta^2}\overline{R}(t)$. Assume that $|r_t| < 1$ for $t \neq 0$. Then from formula (2) we find that for $\beta \to \infty$ and $t \neq 0$, we have

$$\overline{r}(t) \sim \frac{6}{\pi^2} \exp\left[-(2\pi\beta)^2(1-r_t)\right]^* \tag{3}$$

this quantity is already negligibly small for $\beta \geqslant \sqrt{\frac{2}{1-r_t}}$. However, the function $\overline{r}\left(\frac{t}{2\pi\beta}\right)$ (the correlation function of the process $z_\Delta(t) = \frac{\sqrt{12}}{\Delta} \times \psi_\Delta x\left(\frac{t}{2\pi\beta}\right)$) coverges for $\beta \to \infty$, as is indicated by formula (2), to the function

$$\frac{6}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{k^2} \exp\left(r_0'' t^2 k^2\right),$$

* $a \sim b$ denotes that $(a/b) \to 1$.

where $r_0'' = f^2_x$ is the mean square frequency in the spectrum of the process x(t) (the terms $m \neq n$ with in formula (2) drop out).

Here the spectral density of the process $\varphi_\Delta(\lambda)$ will converge to the function

$$\varphi(\lambda) = \frac{6\sqrt{2\pi}}{\pi^2 f_x} \sum_{k=1}^{\infty} \frac{1}{k^3} \exp\left(-\frac{\lambda^2}{2 f_x^2 k^2}\right).$$

Assume the quantization noise $\psi_\Delta x(t)$ is applied to the input of a linear element that converts the signal x(t) to the signal

$$Kx(t) = \int_{-\infty}^{\infty} k(s)\, x(t-s)\, ds,$$

and assume that the transfer function for this system is $k^*(\lambda)$. Then the spectral density of the process $K\psi_\Delta x(t)$ will be close to the function

$$\frac{\Delta^2}{2\pi\beta} |k^*(\lambda)|^2 \varphi\left(\frac{\lambda}{2\pi\beta}\right).$$

For $\beta \to \infty$

$$\varphi\left(\frac{\lambda}{2\pi\beta}\right) \to \varphi(0) = \frac{6\sqrt{2\pi}}{\pi^2 f_x} \sum_{1}^{\infty} \frac{1}{k^3}.$$

Therefore the spectral density of the process $K\psi_\Delta x(t)$ can be assumed equal to

$$C \frac{\Delta^2}{f_x \beta} |k^*(\lambda)|^2 = C \frac{\Delta^2}{\sigma_x} |k^*(\lambda)|^2, \tag{4}$$

where

$$C = \frac{6}{\pi^2 \sqrt{2\pi}} \sum_{1}^{\infty} \frac{1}{k^3} \approx 0{,}3. \tag{5}$$

The error of this approximation can be estimated, for example, as follows. If a linear system does not pass harmonics with frequencies exceeding $f_{K\max}$ (i.e., if $k^*(\lambda) = 0$ for $|\lambda| > f_{K\max}$), then the relative error does not exceed the value

$$1 - \frac{\varphi\left(\frac{1}{2\pi\beta} f_{K\max}\right)}{\varphi(0)} \leqslant \frac{6\sqrt{2\pi}}{2\pi^2 (2\pi)^2 C} \left(\sum_{1}^{\infty} \frac{1}{k^5}\right) \left(\frac{f_{K}\max}{f_x \beta}\right)^2 \approx \left(\frac{0.1 f_{K}\max}{f_x \beta}\right)^2. \tag{6}$$

Thus, for example, if

$$f_K \max \leqslant \beta f_x, \tag{7}$$

then the use of formula (4) leads to an error that does not exceed 1%.

Correspondingly, the correlation function ρ_t of the process $K\psi_\Delta x(t)$ can be assumed equal to*

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* Moreover, it can be proved that the normalized process $(\sqrt{\beta}/\Delta)\, K\psi_\Delta x(t)$ converges to a certain stationary process for $\Delta \to 0$ (or $\beta \to \infty$); thus it is possible to find the probability distribution for it. We do not cite this proof here.

$$\rho_t = C\,\frac{\Delta^2}{\sigma_x}\int\limits_{-\infty}^{\infty} k(s)\,k(t-s)\,ds.$$

Note also that even for $\beta \geq 1$ the quantization noise $\psi_\Delta x(t)$ can be assumed to be uncorrelated with the input process x(t). In fact, in the Appendix we prove [cf. (14a)] that the mutual correlation function for the processes $\psi_\Delta x(t)$ and x(t) is $\overline{R}(t) = Q(\beta)\,\sigma_x^2 r_t$, where $Q(\beta) = \frac{1}{\pi}\sum_{1}^{\infty}\frac{(-1)^k}{k}e^{-2(\pi\beta k)^2}$, and that the quantity $Q(\beta)$ is already negligibly small for $\beta \geq 1$.

2. The Case of a Closed-Loop System

Assume that for passage through a linear element the function x(t) is subjected to the operator L with the transfer function $l^*(\omega)$. Then the equation for determining $y_\Delta(t)$ is written as

$$y_\Delta(t) = L\,[x(t) - \psi_\Delta x(t) - y_\Delta(t) + \psi_\Delta y_\Delta(t)],$$

hence we find that

$$y_\Delta(t) = K\,[x(t) - \psi_\Delta x(t) + \psi_\Delta y_\Delta(t)], \tag{8}$$

where $K = (E + L)^{-1}L$ is a linear operator with the transfer function $k^*(\omega) = \frac{l^*(\omega)}{1 + l^*(\omega)}$ (E is a unit operator); i.e., it is an operator which must be applied to the input signal in order to obtain the output signal for the limiting system

$$y_0(t) = Kx(t). \tag{9}$$

Therefore

$$\delta_\Delta(t) = y_\Delta(t) - y_0(t) = -K\psi_\Delta x(t) + K\psi_\Delta y_\Delta(t). \tag{10}$$

In the right hand side of equation (10) the operation $K\psi_\Delta$ is applied to the processes x(t) and $y_\Delta(t)$. If both of these processes were to be Gaussian, then it would be possible to clarify the properties of the process $\delta_\Delta(t)$ by applying the conclusions of section 1. Although the process $y_\Delta(t)$ is not Gaussian, it is nevertheless close to the Gaussian process $y_0(t)$ for small Δ. It can be proved that for natural assumptions

$$K\psi_\Delta y_\Delta(t) = K\psi_\Delta y_0(t)\,(1 + \alpha_\Delta(t)), \tag{11}$$

where $\alpha_\Delta(t)$ is a stationary process for fixed Δ, and $\alpha_\Delta(t)$ tends toward zero with respect to probability when $\Delta \to \infty$.* Based on relationship (11) we can replace $K\psi_\Delta y_\Delta(t)$ by $K\psi_\Delta y_0(t)$. in (10).

Now making use of the results of section 1, we find that the correlation function for the process can be assumed equal to

$$C\,\frac{\Delta^2}{\sigma_y}\int\limits_{-\infty}^{\infty} k(s)\,k(t-s)\,ds.$$

Thus for a sufficiently small step Δ the error $\delta_\Delta(t)$ can be assumed equal to the result of passing the difference between the quantization noise in the output signal of the limiting system and the input signal through an inertial system:

$$\delta_\Delta(t) \approx K\psi_\Delta y_0(t) - K\psi_\Delta x(t). \tag{10a}$$

* For example, this can be proved for the case where a linear system has a bounded frequency pass band; this condition is practically always satisfied. Under these conditions the quantity $\alpha_\Delta(t)$ has a dispersion of the order of $1/\beta$.

Note that if the normalized mutual correlation function ρ_t for the process x(t) and $y_0(t)$ does not exceed a certain number $\rho < 1$ in absolute magnitude, then the normalized mutual correlation for their quantization noises is already a negligibly small quantity for $\beta \geqslant \sqrt{\frac{2}{1-\rho}}$. In fact, by analogy with relationship (3) it is easy to find that for $\beta \to \infty$ the mutual correlation function $\bar{\bar{\rho}}_t = \frac{12}{\Delta^2} M\psi_\Delta x(0)\,\psi_\Delta y_0(t)$ satisfies the relationship

$$\bar{\bar{\rho}}_t \sim \frac{6}{\pi^2}\exp\{-(2\pi\beta)^2(1-\rho_t)\} \quad (-\infty < t < \infty)$$

and is a negligibly small quantity for $\beta \geqslant \sqrt{\frac{2}{1-\rho}}$. In that case the processes $K\psi_\Delta y_0(t)$ and $K\psi_\Delta x(t)$ can be assumed uncorrelated, and the correlation function for the process $\delta_\Delta(t)$ can be assumed equivalent to the sum of the correlation functions for the processes $K\psi_\Delta y_0(t)$ and $K\psi_\Delta x(t)$. Applying the conclusions of section 1 to these processes, we find that the correlation function for the process $\delta_\Delta(t)$ can be assumed equal to

$$C\Delta^3\left(\frac{1}{\sigma_x'}+\frac{1}{\sigma_y'}\right)\int_{-\infty}^{\infty} k(s)\,k(t-s)\,ds.$$

If we do not assume that $\beta \geqslant \sqrt{\frac{2}{1-\rho}}$, then from relationship (10a) it is possible to find the estimate of the dispersion for the process according to the formula

$$D\delta_\Delta(t) \leqslant \left(\sqrt{D[K\psi_\Delta y_0(t)]}+\sqrt{D[K\psi_\Delta x(t)]}\right)^2 = C\Delta^3\left(\frac{1}{\sqrt{\sigma_x'}}+\frac{1}{\sqrt{\sigma_y'}}\right)^2\int_{-\infty}^{\infty} k^2(s)\,ds.$$

SUMMARY

1. If the input of a digital automatic system is subjected to a Gaussian stationary process x(t) and the width $f_{K\max}$ of the frequency band passed by the linear element does not exceed the mean frequency $\bar{f}$ of the input signal multiplied by β, then the quantization noise $\psi_\Delta x(t)$ can be treated as white noise; i.e., it can be treated as a process with a constant spectral density $0.3(\Delta^3/\sigma_x')$ which is not correlated with the signal x(t).

Here the correlation process for the function $\delta_\Delta(t) = K\psi_\Delta x(t)$; can be assumed equal to

$$\rho_t = 0{,}3\,\frac{\Delta^3}{\sigma_x'}\int_{-\infty}^{\infty} k(s)\,k(t-s)\,ds.$$

2. In the case of a closed-loop system and identical conditions it can be assumed that the dispersion of the process $\delta_\Delta(t) = y_\Delta(t) - y_0(t)$ is

$$D\delta_\Delta(t) \leqslant 0{,}3\Delta^3\left(\frac{1}{\sqrt{\sigma_x'}}+\frac{1}{\sqrt{\sigma_y'}}\right)^2\int_{-\infty}^{\infty} k^2(s)\,ds.$$

If under these conditions the normalized mutual correlation function for the input signal x(t) and the output signal $y_0(t) = Kx(t)$ of the limiting system does not exceed $\rho < 1$ and the condition $\beta \geqslant \sqrt{\frac{2}{1-\rho}}$, is valid, then the correlation function for the process $\delta_\Delta(t)$ can be assumed equal to

$$0{,}3\Delta^3\left(\frac{1}{\sigma_x'}+\frac{1}{\sigma_y'}\right)\int_{-\infty}^{\infty} k(s)\,k(t-s)\,ds.$$

APPENDIX

The Correlation Function for the Level Quantization Noise in the Case of a Gaussian Process, and the Mutual Correlation Function for the Process Plus the Noise

Assume (ξ, η) is a pair of Gaussian random functions with zero mathematical expectation and a second-moment matrix

$$\begin{Vmatrix} \sigma^2, & r\sigma^2 \\ r\sigma^2, & \sigma^2 \end{Vmatrix}.$$

We shall find the quantities $\overline{R} = M\,[\xi, \psi_\Delta \eta]$ and $\overline{\overline{R}} = M\,[\psi_\Delta \xi, \psi_\Delta \eta]$ [cf. (1)]. We shall introduce the substitution $\beta = \sigma/\Delta$.

Assume p(x, y) are the distribution densities for the vectors (ξ, η). Then the distribution densities for the vectors $(\psi_\Delta \xi, \psi_\Delta \eta)$ and $(\xi, \psi_\Delta \eta)$ will respectively be equal to

$$\overline{\overline{p}}(x, y) = \sum_{j,k} p(x + j\Delta, y + k\Delta), \quad |x|, |y| \leq \frac{\Delta}{2},$$

$$\overline{p}(x, y) = \sum_{k} p(x, y + k\Delta), \quad |y| \leq \frac{\Delta}{2}.$$

Assume now that $\varphi(\lambda, \omega)$ is the characteristic function of the vector (ξ, η); i.e.,

$$\varphi(\lambda, \omega) = M\,[\exp\{i(\lambda\xi + \omega\eta)\}] = \exp\left\{-\frac{\sigma^2}{2}(\lambda^2 + 2r\lambda\omega + \omega^2)\right\}. \tag{12}$$

Then

$$\overline{\overline{p}}(x, y) = \sum_{m,n} a_{mn} \exp\left\{-i\frac{2\pi}{\Delta}(mx + ny)\right\},$$

where

$$a_{mn} = \frac{1}{\Delta^2}\varphi\left(\frac{2\pi}{\Delta}m, \frac{2\pi}{\Delta}n\right)$$

and

$$\overline{p}(x, y) = \sum_{k} \int_{-\infty}^{\infty} a_k(\lambda)\, e^{-i\lambda x} d\lambda\, e^{-i\frac{2\pi}{\Delta}ky},$$

where

$$a_k(\lambda) = \frac{1}{2\pi\Delta}\varphi\left(\lambda, \frac{2\pi}{\Delta}k\right).$$

From this we obtain

$$\overline{\overline{R}} = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} xy\overline{\overline{p}}(x, y)\, dx dy = \left(\frac{\Delta}{2\pi}\right)^2 \sum_{m,n \neq 0} \frac{(-1)^{m+n+1}}{mn}\varphi\left(\frac{2\pi}{\Delta}m, \frac{2\pi}{\Delta}n\right), \tag{13}$$

$$\overline{R} = \int_{-\infty}^{\infty} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} xy\overline{p}(x, y)\, dy dx = -\frac{\Delta}{2\pi}\sum_{k \neq 0} \frac{(-1)^k}{k}\frac{\partial}{\partial\lambda}\varphi\left(0, \frac{2\pi}{\Delta}k\right). \tag{14}$$

Making use of the expression (12) for the function $\varphi(\lambda, \omega)$ and performing simplifications, we obtain*

$$\overline{\overline{R}} = \left(\frac{\Delta}{\pi}\right)^2 \sum_{m,n=1}^{\infty} \frac{(-1)^{m+n}}{mn} \exp\left\{-(2\pi\beta)^2 \frac{m^2+n^2}{2}\right\} \operatorname{sh}(2\pi\beta)^2 rmn], \tag{13a}$$

$$\overline{R} = Q(\beta)\,\sigma^2 r, \tag{14a}$$

where

$$Q(\beta) = -2 \sum_{k=1}^{\infty} (-1)^k e^{-2(\pi\beta k)^2}.$$

In particular, if $\xi = x(0)$, $\eta = x(t)$, where x(t) is a stationary Gaussian process, it follows that $r = r_t$ will be the normalized correlation function for the process x(t). Under these conditions formulas (13a) and (14a) yield the following expressions for $\overline{\overline{R}} = \overline{\overline{R}}(t)$ (the unnormalized correlation function for the process $\psi_{\Delta}x(t)$), and $\overline{R} = \overline{R}(t)$ (the mutual correlation function for the processes x(t) and $\psi_{\Delta}x(t)$).

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2. B. R. Levin, The Theory of Random Processes and Its Application in Radio-Engineering [in Russian] (Soviet Radio Press, 1960).

All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. *Some or all of this periodical literature may well be available in English translation.* A complete list of the cover-to-cover English translations appears at the back of this issue.

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* In contrast to [2] the expression for the quantity r is derived without using Fourier transforms [cf. [2], formula (7.124)].

SYNTHESIS METHOD FOR FINITE AUTOMATA

V. G. Lazarev and E. I. Piil'

(Moscow)

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A synthesis method is presented for discrete circuits with feedback, containing the operators AND, OR, NOR, d (transition operator), and feedback elements (relays, triggers, delay lines, etc.). In this connection selection criteria for various types of feedback are examined.

In automation, remote control, communications, computer technology, multisequence circuits (finite automata) constructed from contactless elements find a wide application. To synthesize these circuits logical devices are widely used [1-5].

It is possible to represent multisequence circuits by oriented (n, k) poles with s feedbacks, where n is the number of inputs, k the number of outputs. In the case where combinations of input signals uniquely determine combinations of output signals, there is no feedback. Then it is possible to reduce the construction of the circuit to the synthesis of an (n, k) pole with operators AND, OR, NOR, if the input and output signals are of the same type (potentials or impulses). If, however, the input signals are potentials and the output impulses then, besides operators AND, OR, NOR, the transition operator d is used [6].

In the case when combinations of output signals are not uniquely determined by combinations of input signals it is necessary to introduce additional signals at the input end to satisfy the given conditions. These additional signals can be obtained with the help of feedback loops. Then, we introduce the necessary number s of these feedbacks and build the (n + s, k + s) pole, where the outputs n + 1, . . . , n + s, are the inputs to the corresponding feedback elements (FE) and the additional inputs n + 1, . . . , n + s are the outputs of FE.*

The number of necessary feedback signals is determined by both the given operating conditions of the circuit and the inherent properties of the feedback schemes.

We shall examine those FE which give rise to potential signals at the output. Then, depending on the nature of the inputs, the FE can be divided into two basic groups:

1) FE reacting only to combinations of input signals,

2) FE reacting to signals corresponding to some transition from one combination of inputs to another.

FE in the first group are called FE of type A (FE-A); FE in the second group FE of type B (FE-B).

Considering that FE introduce a delay τ ($\tau < T_{min}$, where T_{min} is the minimum duration of the combinations of input signals), we shall examine such FE-A the output signals of which repeat the input signals with delay τ, and such FE-B the output signals of which appear with delay τ after the appearance of the switching-in signal at the input and vanish in time τ after the appearance at the input of the switching-out signal. Electromechanical unpolarized relays and delay lines are examples of FE-A; thyratrons and triggers are examples of FE-B.

Different types of FE require different approaches for switching them at proper moments into the circuit operation for the purpose of transforming unrealizable conditions to realizable ones.

FE-A have been studied by many authors [1, 4, 7-11]. Estimates of the minimal necessary number of FE have been obtained [8, 9, 11] and conditions for realizing the switching tables have been formulated [10].

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* Input and output signals of FE correspond to one and the same ordinal number, therefore the additional output signals have the numbers n + 1, . . . , n + s and not k + 1, . . . , k + s.

All these problems have not been sufficiently studied for applications to FE-B, although the latter are widely used in practice.

The present paper formulates the conditions for switching FE-B into the circuit operation, gives an estimate for the minimum number of them for realizing the conditions, and proposes ways to synthesize finite automata using FE-B.

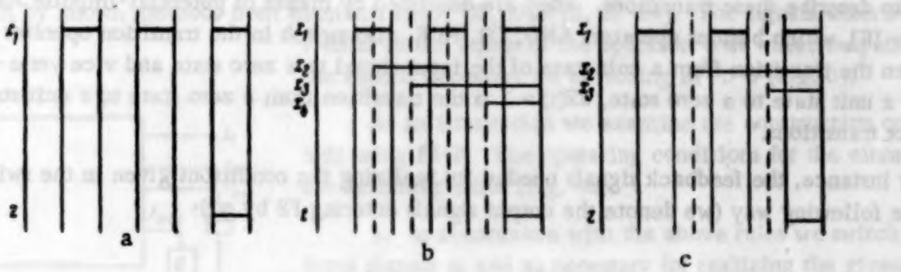

Fig. 1

We assume that the operating conditions of the finite automata are given in the form of switching tables [7, 10]. Then to transform unrealizable switching tables to realizable ones it is necessary to switch in s FE.

If we use FE reacting to a combination of input signals (FE-A), they can be switched-in independently of previous switching-ins in sequence with such combinations of input signals which did not occur in the previous sequence.

However, if we use FE reacting to transitions from one combination of input signals to another (FE-B), they can be switched-in independently of previous switching-in signals, produced by the transitions, not occurring earlier.

Since for a given number $n + s$ of input signals the number of different transitions can be greater than the number of combinations, then the number of sequences where it is possible to switch in FE-B is greater than the number of sequences where it is possible to switch in FE-A.

For instance, for $n + s$ input signals the number of combinations can be a maximum of 2^{n+s}, however, the number of transitions may be $(2^{n+s}-1)2^{n+s}$ if coincidence of transitions is allowed, i.e., if simultaneous changes of two or more input signals are permitted; and, the number of transitions is $(n + s)2^{n+s}$ when coincidence of transitions is not allowed.

Therefore, in a number of cases the use of FE-B makes it possible to reduce the number of FE necessary for transforming unrealizable conditions to realizable ones.

For instance, let the switching table* be given as shown in Fig. 1a.

Then, if we use FE-A to transform the unrealizable switching table (Fig. 1a) to a realizable one it is necessary to use three FE-A (Fig. 1b); however, if we use FE-B then only two FE-B are necessary (Fig. 1c).

However, we have not succeeded in changing the estimate previously obtained [11] for the necessary number of FE. In fact, if there exists m_{max} sequences with identical states of the input signals to every one of which for given conditions there must correspond different combinations of output signals (coincident sequences), then to distinguish between these sequences it is necessary to introduce not less than 2^{s_N} states of the secondary input signals. Here s_N is the lower estimate of the necessary number of secondary signals (or the number of FE).

We have also not succeeded in decreasing the upper estimate, because in the case when $m_1 = m_2 = \ldots = m_{max}$, when using FE-B we get for the upper estimate [11]:

$$2^{s_u} \geq 2\,m_{max} - 1.$$

* In the switching table we use the following notation convention: x_i are the input signals, z_i are the output signals, ━ denotes the presence of a signal at the fundamental input, ─ denotes the presence of a signal at the secondary input or at the output.

Thus, without lowering the estimate of the number of FE, the use of FE-B makes it possible to reduce the number of cases in which the upper estimate is realized.

In those cases when the number of necessary feedback signals cannot be decreased, it is possible to reduce the number of states of these signals we need to use, i.e., a smaller number of their switching-ins and switching-outs.

If we use FE-B we change to method of recording the signals entering FE. Since the output signals of the circuit which are the inputs of the FE are produced by the transition of one combination of input signals to another, then it is necessary to describe these transitions. They are described by means of potential-impulse forms, introduced by A. D. Talantsev [6], which besides operators AND, OR, NOR, also switch in the transition operator d. Here we distinguish between the transition from a unit state of the input signal to a zero state and vice versa: $dx(t) = 1$ is the transition from a unit state to a zero state, $d\bar{x}(t) = 1$ is the transition from a zero state to a unit state, $dx(t) = 0$ and $d\bar{x}(t) = 0$ are not transitions.*

Thus, for instance, the feedback signals needed for realizing the conditions given in the switching table (Fig. 2) is written in the following way (we denote the output signals entering FE by y_i):

$$y_3 = \bar{x}_2 \bar{x}_3 \bar{x}_4 \bar{x}_5 \, dx_1 \vee \bar{x}_1 x_3 x_4 \bar{x}_5 \, dx_2,$$
$$y_4 = \bar{x}_1 x_3 \bar{x}_4 \bar{x}_5 \, dx_2 \vee \bar{x}_1 \bar{x}_3 x_4 x_5 \, dx_2,$$
$$y_5 = \bar{x}_1 \bar{x}_3 x_4 \bar{x}_5 \, d\bar{x}_2.$$

The output signals of the fundamental outputs, having a potential character, are written in the usual form [3, 10]:

$$z_1 = \bar{x}_1 x_2 x_3 \bar{x}_4 \bar{x}_5 \vee \bar{x}_1 x_2 x_3 x_4 x_5 \vee \bar{x}_1 \bar{x}_2 x_3 x_4 \bar{x}_5 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 \bar{x}_5 \vee x_1 \bar{x}_2 \bar{x}_3 x_4 x_5,$$
$$z_2 = \bar{x}_1 x_2 \bar{x}_3 x_4 \bar{x}_5 \vee \bar{x}_1 x_2 \bar{x}_3 x_4 x_5 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 x_5 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 x_5.$$

The method of constructing circuits to obtain potential signals is well known [2, 12-14]. It would be desirable to express the signals for switching in FE-B in the usual form to construct the entire circuit, in the first place to make use of already known methods, and in the second place to obtain a unified circuit for all k + s outputs of the (n + s, k + s) pole. Such unification of circuits permits the reduction to the smallest number of operators AND, OR, and NOR.

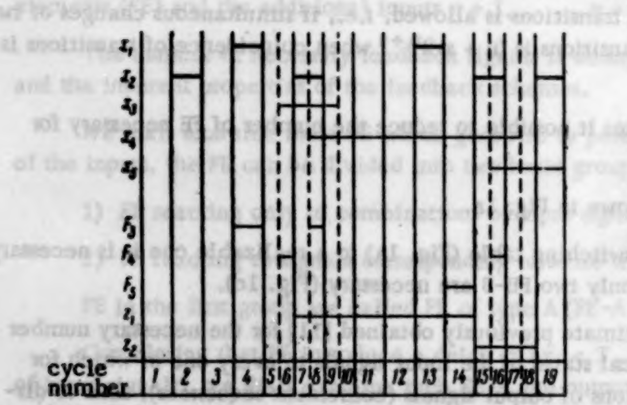

Fig. 2

Signals to switch in FE-B can be described by Boolean functions if we can find such a function F that $y = dF$, i.e., if there exists such a potential function F which after differentiation gives the necessary impulse signal y. Here the circuit takes the form shown in Fig. 3. Here every signal except y is a potential.

The process of finding the function F is called integration of the potential-impulse form y [6, 15].

Function F can be obtained immediately from the operating conditions of the circuit given in the tables. In fact, for instance, if in the transition from the fourth to the fifth sequence (Fig. 2) it is necessary to obtain the signal for switching in FE-B, then the function F_3 in the fifth sequence must necessarily have the value zero, and in the preceding fourth sequence must necessarily have the value one (if we use the transition from the unit to the zero state; analogously it is possible to use the inverse transition and to get the function F).

* For conciseness the parameter t will be omitted in what follows.

Thus, for our example function F_3 can be written as

$$F_3 = x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \bar{x}_5 \vee \bar{x}_1 x_2 x_3 x_4 \bar{x}_5.$$

The state of the input signals at which the function F_3 must necessarily have unit and zero values is defined ($F_3 = 0$ at $x_1x_2x_3x_4x_5$ and $x_1x_2x_3x_4x_5$).

Thus we can obtain function F written in the same form as the fundamental output signal z, and therefore can construct the circuit by known methods from elements AND, OR, NOR [2, 12-14]. The signals described by function F enter at the inputs of the operators d at whose outputs are obtained the signals necessary for switching in $\overline{\text{FE}}$-B (Fig. 3).

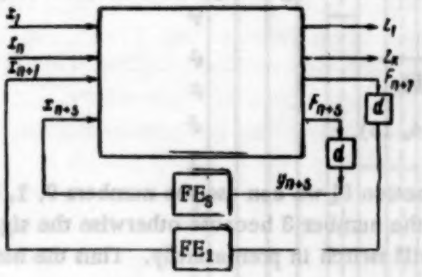

Fig. 3

As an illustration we examine the construction order for a circuit using FE-B. The operating conditions for the circuit are given in the switching table (Fig. 4a).

1. In accordance with the above rules we switch in the secondary input signals x_3 and x_4 necessary for realizing the given conditions (Fig. 4b).

2. Having assigned weights to the input signals: x_1-1, x_2-2, x_3-4, x_4-8, we obtain the necessary and conditional selection of numbers for the output signals z_1 and z_2 [10]:

$$z_1 = 3\ (2, 6, 7, 10, 14, 15),$$
$$z_2 = 11, 13\ (2, 6, 7, 10, 14, 15).$$

Here the conditional numbers are the unused numbers.

3. We consider two cases when for FE-B we use: a) triggers with separate inputs, and b) triggers with denumerable inputs.

We obtain the necessary and conditional numbers for case (a). The signals reacting on the trigger must be separated into two groups: signals switching in the trigger (y'), and signals switching out the trigger (y"). We can determine the corresponding functions F' and F".

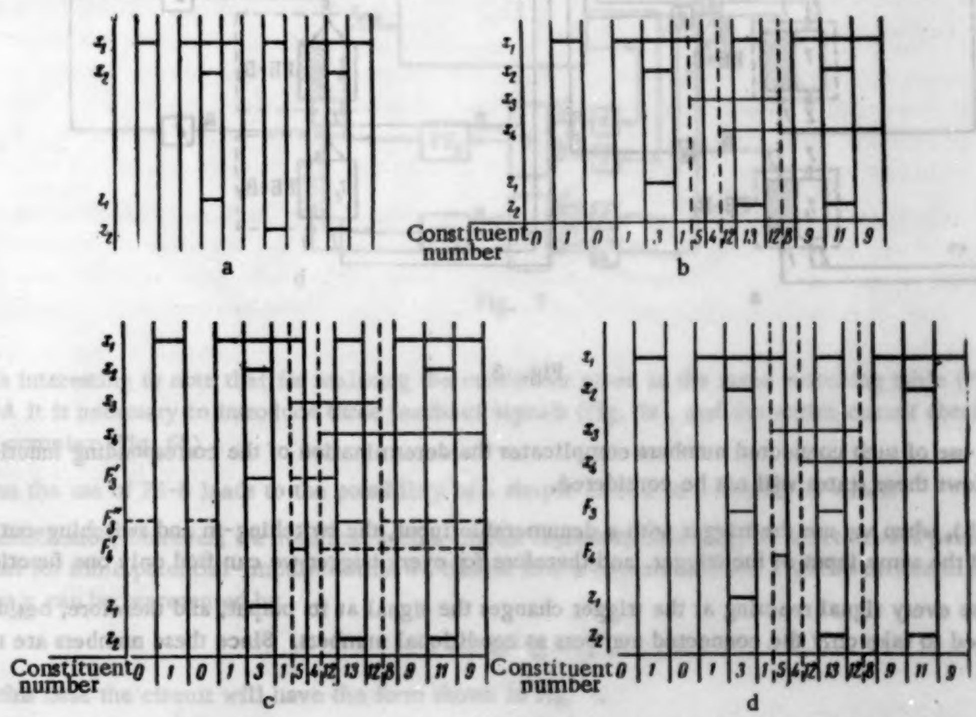

Fig. 4

From the table in Fig. 4c we find the necessary number of functions F_3':

$$F_3' = 3.$$

Besides the unused ones, the numbers 4, 5, 12 are also conditional numbers for F_3' because the appearance of signal y_3' in any sequence up to the moment the signal y_3'' appears does not change the signal at the output of this trigger.

Thus,

$$F_3' = 3\ (2,\ 4,\ 5,\ 6,\ 7,\ 10,\ 12,\ 14,\ 15).$$

Analogously we can show the equivalent states for functions F_3'' and F_4' (since the signal x_4 is not switched out, F_4'' is not defined).

$$F_3'' = 13\ (0,\ 2,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 14,\ 15),$$
$$F_4' = 5\ (2,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15).$$

Besides the numbers 8, 9, 11, 12, 13 as conditional numbers for function F_4' we can use the numbers 0, 1, 3. But, when considering the necessary number 1 it is also necessary to use the number 3 because otherwise the signal y_4' will appear at the transition from state 1 to state 3, i.e., the trigger will switch in prematurely. Thus the numbers 0, 1, 3 are interconnected.

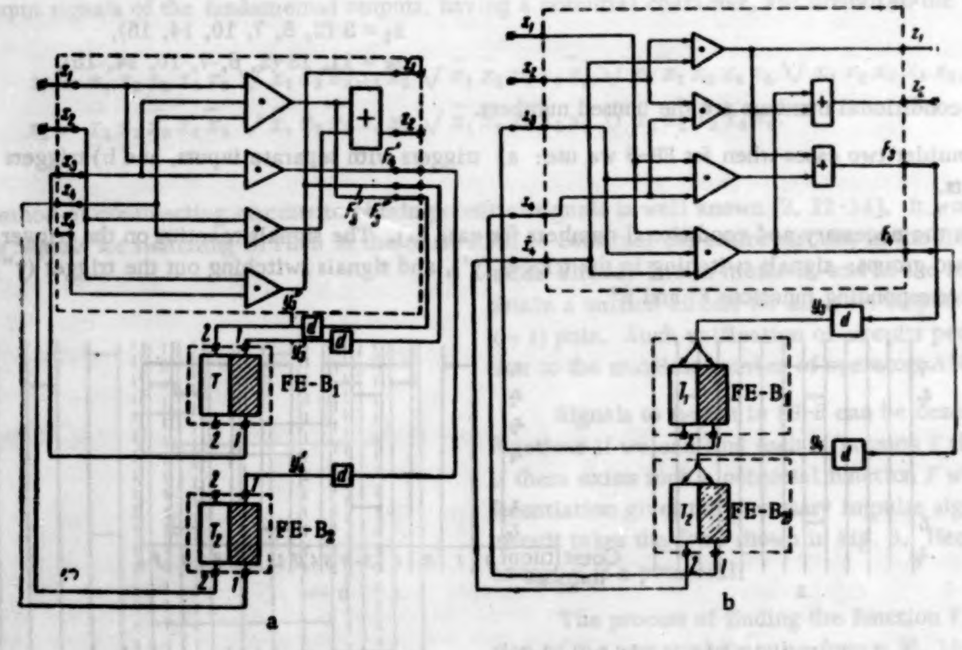

Fig. 5

Since the use of such connected numbers complicates the determination of the corresponding function F, here and in what follows these states will not be considered.

For case (b), when we use the trigger with a denumerable input, the switching-in and switching-out signals enter at one and the same input of the trigger, and therefore for every trigger we can find only one function F (Fig. 4d).

In this case every signal reacting at the trigger changes the signal at its output, and therefore, besides the unused ones we need to take only the connected numbers as conditional numbers. Since these numbers are not to be used, then

$F_3 = 3, 13\ (2, 6, 7, 10, 14, 15)$
$F_4 = 5\ (2, 6, 7, 10, 14, 15).$

4. In each of the two cases we simplify the determination of the functions z and F with the use of the conditional numbers [10] and then construct the circuit from the elements AND, OR, NOR, and d. Figure 5a shows the circuit realizing the conditions given in the table of Fig. 4a for case (a), and Fig. 5b for case (b).*

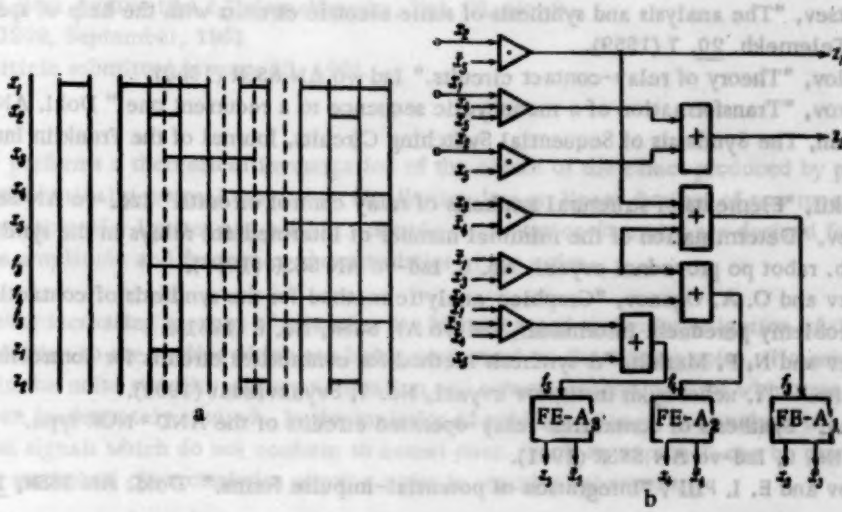

Fig. 6

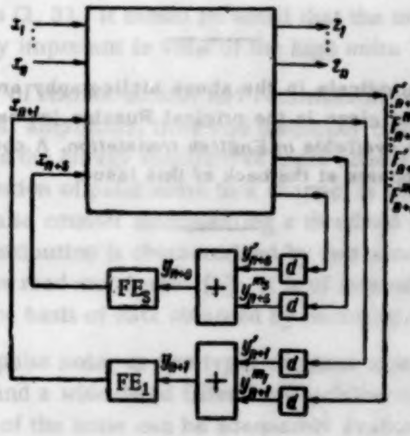

Fig. 7

It is interesting to note that for realizing the conditions given in the same switching table (Fig. 4a), when using FE-A it is necessary to introduce three feedback signals (Fig. 6a), and the entire circuit obtained becomes considerably complex (Fig. 6b).

Thus the use of FE-B leads to the possibility of a simple circuit in a number of cases.

In conclusion, however, we note that such a circuit representation (Fig. 3) is not always possible. It is known [6, 15] that for some potential-impulse forms we cannot find a unique function F whose differential equals y. Here sometimes y can be represented by

$$y = \vee\, dF^j \quad (j = 1, \ldots, m).$$

In this case the circuit will have the form shown in Fig. 7.

* **In the diagrams we represent the operator OR (realizing Boolean addition) by the symbol + in a rectangle, and the operator AND (realizing Boolean multiplication by the symbol • in a triangle.**

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THE EFFECT OF PULSE NOISE ON TELEMETERING UNITS

L. B. Venchkovskii

(Moscow)

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The paper performs a theoretical investigation of the nature of the effect produced by pulse noise with a logarithmically-normal amplitude distribution law on linear devices of two types and on a system consisting of a limiter and a filter. Simple computation formulas are derived for determining the amplitude and frequency characteristics of the noise.

Recently an ever increasing amount of attention has been devoted to the investigation of the interfering effect of pulse noise, in addition to the studies which are being performed on fluctuating noise. However, heretofore the problems involved in the noise stability of communication and remote control systems which are subjected to actual pulse noise have been inadequately studied. In the majority of published papers the analysis has been performed for idealized pulse noise signals which do not conform to actual ones. Such an approach can be explained by the absence of adequate data on statistical characteristics of pulse noise in practical channels.

In addition, the investigation of noise performed by a number of authors [1,2] has made it possible to conclude that many types of industrial noise can be included in the class of pulse noise signals with a logarithmically-normal amplitude distribution law. This type of noise exists, in particular, in low-voltage power nets (0.4/6 kv) which are used for a number of industrial objects [1, 3]. It should be noted that the noise stability for transmitting information over such channels becomes especially important in view of the high noise level and the large attenuation.

In computing the noise stability of remote control and communication systems when pulse noise is present it is necessary to determine the statistical amplitude, time and frequency characteristics for the noise output of various linear and nonlinear devices, as well as the energy spectrum of these noise signals. As was demonstrated in [1], the measurement of the amplitude distribution of pulse noise in a channel is conveniently performed using a selector unit with an inertial detector and a pulse counter incorporating a threshold relay. For noise signals with a logarithmically-normal law the amplitude distribution is characterized by two parameters which can be determined by processing the data at the output of the read-out device [1]. It is of interest to develop methods for computing the noise stability of various devices on the basis of data obtained by means of the read-out device.

The paper studies the effect of pulse noise on two types of linear units: a narrow-band filter at whose output the noise becomes fluctuating noise, and a wide-band filter for which the noise at the output becomes pulse noise. In the first case the interfering effect of the noise can be adequately evaluated on the basis of the noise power; in the second case it is necessary to determine the parameters of the amplitude distribution as well.

We shall study a nonlinear device consisting of an inertialess limiter; we shall develop a method for computing a system consisting of a filter, a limiter, and another filter. The analysis of the passage of pulse noise through the enumerated devices requires a more detailed study of the amplitude characteristics for logarithmically-normal distribution. We have derived formulas for computing the distribution moments, as well as the expression for the characteristic function.

1. The Parameters of Logarithmically-Normal Distribution

The distribution of the amplitude for pulse noise signals in the class indicated above can be written conveniently on a logarithmic scale (Fig. 1). For u = ln x

$$\varphi(u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(u-\bar{u})^2}{2\sigma^2}}, \tag{1}$$

where $u = \ln a$ and σ respectively denote the average and mean-square values of the distribution $\underline{u}$. The parameters $\bar{\underline{u}}$ and σ fully characterize the distribution $\underline{u}$. As we shall demonstrate below, all of the basic characteristics of the distribution $\underline{x}$ can be expressed in terms of these two parameters.

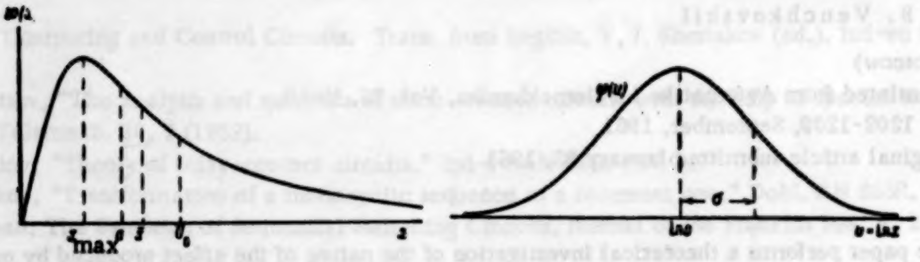

Fig. 1

It is not difficult to show that the quantity σ is independent of the units in which the noise amplitude is measured and is a dimensionless quantity. The quantity $\bar{\underline{u}}$ is dimensional, and when the scale varies by a factor of $\underline{k}$ the value of $\underline{u}$ varies by the amount $\ln k$. Note that the difference $u-\bar{u}$, just as σ, is dimensionless.

In the case where we study a logarithmic distribution with the base $\underline{b}$ instead of natural logarithms, the average and mean-square values are increased by a factor of $\log_b e$ and the abscissas of the distribution are reduced by the same factor.

Making use of the conventional laws for nonlinear transformation of the distribution law, we find the distribution for $\underline{x}$

$$w(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln x - \ln a)^2}{2\sigma^2}} \tag{2}$$

The characteristic points on this distribution (Fig. 1) are the median which is equal to $\underline{a}$, and the abscissa of the most probable value x_{max} (the mode). From the condition $w'(x) = 0$ we obtain

$$x_{max} = a e^{-\sigma^2} \tag{3}$$

The moments for the distribution $\underline{x}$ can be determined conveniently in terms of the derivatives of the characteristic function which is equal to

$$\theta_x(v) = m_1(e^{ive^u}) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{ive^u} e^{-\frac{(u-\bar{u})^2}{2\sigma^2}} du \tag{4}$$

for a logarithmically-normal law.

The $\underline{k}$th order moment which is determined from the condition

$$\left[\frac{d^k \theta_x(v)}{dv^k}\right]_{v=0} = i^k m_k\{x\}, \tag{5}$$

is equal to

$$m_k = a^k e^{\frac{(k\sigma)^2}{2}}. \tag{6}$$

The distribution moments of the first two orders are of greatest significance in describing the distribution $\underline{x}$; these are equal to

$$m_1\{x\} = \bar{x} = ae^{\sigma^2/2}, \tag{7}$$

$$m_2\{x\} = a^2 e^{(2\sigma)^2/2}. \tag{8}$$

The dispersion of the distribution is written as

$$M_2\{x\} = D_x = \sigma_x^2 = a^2 e^{\sigma^2}(e^{\sigma^2} - 1). \tag{9}$$

For certain special applications it is convenient to make use of a characteristic function which is written in the form of a MacLauren series in accordance with (5):

$$\theta_x(v) = \sum_{k=0}^{\infty} \frac{m_k}{k!}(iv)^k = \sum_{k=0}^{\infty} \frac{(iva)^k}{k!} e^{\frac{(k\sigma)^2}{2}}. \tag{10}$$

2. Parameters of the Clipped Distribution

In analyzing the passage of pulse noise through nonlinear devices of the inertialess limiter type it is necessary to determine the parameters of the clipped distribution. We shall determine the first two distribution moments for pulses which have passed through a limiter which limits top and bottom at the level U_0 (Fig. 1).

The average value of the amplitudes for the peaks exceeding U_0 can be determined from the formula

$$\bar{x}_b = \frac{\int_{U_0}^{\infty} \exp\left[-\frac{(\ln x - \ln a)^2}{2\sigma^2}\right] dx}{\int_{U_0}^{\infty} \frac{1}{x} \exp\left[-\frac{(\ln x - \ln a)^2}{2\sigma^2}\right] dx}. \tag{11}$$

We shall introduce the relative cutoff level

$$\alpha = \frac{\ln U_0 - \ln a}{\sigma}. \tag{12}$$

Substituting (12) and (7) into (11), we obtain

$$\bar{x}_b = \frac{\bar{x} V(\alpha - \sigma)}{V(\alpha)}, \tag{13}$$

where $V(\alpha)$ is the tabulated function

$$V(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{t^2}{2}} dt. \tag{14}$$

In analogous fashion we find the average amplitude of those pulses which do not exceed U_0:

$$\bar{x}_M = \frac{\bar{x}[1 - V(\alpha - \sigma)]}{1 - V(\alpha)}. \tag{15}$$

The second order moment is found from expression (2). For pulses above the threshold

$$m_{2b} = \frac{m_2 V(\alpha - 2\sigma)}{V(\alpha)}. \tag{16}$$

For pulses below the threshold

$$m_{2M} = \frac{m_2[1 - V(\alpha - 2\sigma)]}{1 - V(\alpha)}. \tag{17}$$

The dispersion of the clipped distribution of the noise peak amplitudes is equal to

$$D_b = m_{2b} - m_{1b}^2 = \frac{m_2 V(\alpha - 2\sigma)}{V(\alpha)} - \frac{m_1 V^2(\alpha - \sigma)}{V^2(\alpha)} \tag{18}$$

and

$$D_M = m_{2M} - m_{1M}^2 = \frac{m_2[1 - V(\alpha - 2\sigma)]}{1 - V(\alpha)} - \frac{m_1^2[1 - V(\alpha - \sigma)]^2}{[1 - V(\alpha)]^2} \tag{19}$$

respectively.

3. The Effect of Pulse Noise on Linear Devices

We shall study the case for which brief pulses of arbitrary but identical shape with a random instant of appearance and a random amplitude u_n exist in a channel. We shall assume that the distribution function for the pulse amplitudes obeys a logarithmically-normal distribution law, and that the spectral density for a pulse of unit amplitude is equal to $G(\omega)$.

Assume that the noise signals in the channel are measured by a selective read-out device with a frequency response $f(\omega)$* and with a pass band such that superposition of pulses at the output can be neglected. The amplitude of the noise peak at the output of the read-out device is related to the amplitude of the peak in the channel by a simple expression

$$x = \frac{u}{2\pi} \int_{-\infty}^{\infty} G(\omega) f_r(\omega)\, d\omega. \tag{20}$$

If within the limits of the pass band of a selective read-out device the spectral density $G(\omega)$ is constant and equal to $G(\omega_0)$ (where ω_0 is the tuned frequency of the read-out device), then

$$x = G(\omega_0)\, \Delta f_r\, u_1. \tag{21}$$

(It is assumed that the read-out device is graduated in such a way that its gain can be assumed equal to unity.)

It follows from formula (21) that the amplitude of the peak at the output of the read-out device proves to be proportional to the equivalent pass band Δf_r of the read-out device which is expressed as

$$\Delta f_r = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_r(\omega)\, d\omega. \tag{22}$$

Assume that we know the parameters of the amplitude distribution $\bar{u}$ and σ which are determined at the output of the read-out device. We shall demonstrate how these parameters vary when a random pulse process passes through a linear device.

Assume that a receiver with an input filter which has a frequency response $z(\omega)$ that differs from the response of the read-out device but is such that pulses at the filter output do not overlap is connected to a communication channel. The transfer coefficient for the filter is assumed equal to k_0. The amplitude of the noise at the output of such a filter when a pulse u_p is applied to its input is equal to

$$u_f = k_0 \gamma' x, \tag{23}$$

where

$$\gamma' = \frac{\int_{-\infty}^{\infty} G(\omega)\, z(\omega)\, d\omega}{\int_{-\infty}^{\infty} G(\omega)\, f(\omega)\, d\omega}.$$

* The frequency responses are assumed to be normalized.

Since the factor multiplying x does not depend on the amplitude of the peak we can conclude that for linear devices the amplitude distribution law for the noise remains the same when the individual pulses do not overlap. The distribution parameters can be expressed as follows in accordance with (2):

$$\ln u_f = \ln a + \ln k_0 \gamma', \tag{24}$$

$$\sigma_f = \sigma. \tag{25}$$

If the spectral density of the noise is uniform within the limits of the pass bands of both the receiver input filter and the read-out device, then the quantity γ is equal to the ratio between the equivalent pass bands of the receiver and read-out device and

$$u_f = \frac{x k_0 \Delta f_f}{\Delta f_r}, \tag{26}$$

where Δf_f is the equivalent pass band of the receiver filter

$$\Delta f_f = \frac{1}{2\pi} \int_{-\infty}^{\infty} z(\omega)\, d\omega. \tag{27}$$

As an example of the second case of linear transformation we shall study a narrow-band filter with a frequency response $z(\omega)$ such that the noise at its output can be treated as fluctuating noise. Under these conditions it is expedient to evaluate the noise signals according to their power.

The energy spectrum for the noise at the filter output is equal to

$$F(\omega) = W(\omega)\, z^2(\omega), \tag{28}$$

where $W(\omega)$ is the energy spectrum for the noise in the channel.

We know [4] that the energy spectrum for a chaotic train of pulses that are of identical shape but of different amplitude is given by

$$W(\omega) = 2n\, \overline{|u_p G(\omega)|^2} = \frac{2n\, \overline{x^2}}{\Delta f_r^2}, \tag{29}$$

where n is the average number of peaks per second. Substituting (29) into (28) and integrating within the limits of the filter pass band, we obtain the following expression for the noise power at the filter output:

$$P_{pf} = \frac{2n\, \Delta F_{ef}\, \overline{x^2}}{\Delta f_r^2}. \tag{30}$$

After substituting (8) and (30), we obtain

$$P_{pf} = \frac{2n\, \Delta F_{ef}\, a^2 e^{2\sigma^2}}{\Delta f_r^2}, \tag{31}$$

where ΔF_{ef} is the energy width of the filter pass band:

$$\Delta F_{ef} = \frac{1}{2\pi} \int_{-\infty}^{\infty} z^2(\omega)\, d\omega. \tag{32}$$

For the same conditions as those which apply in formula (26) we obtain the expression for the noise power at the filter output:

$$P_{pf} = P_{pr} \frac{\Delta F_{ef}}{\Delta f_{er}}, \tag{33}$$

where P_{pr} and Δf_{er} respectively denote the noise power at the output of the read-out device and its energy band width.

Note that in scaling the amplitude characteristics of the pulse noise we use the ratio between the equivalent pass bands of the filter and the read-out device; in computing the power we use the ratio between the energy (effective) bands. Thus, in computing the passage of pulse noise through linear devices it is necessary to know the parameters of the amplitude distribution at the output of the selective read-out device, as well as the equivalent and effective pass bands of the read-out device.

4. The Passage of Pulse Noise through a Device with a Nonlinear Element

We shall study the effect of pulse noise with a logarithmically-normal amplitude distribution law on a receiver consisting of a relatively wide-band filter 1 with a pass band Δf_f and a gain k_0, an inertialess detector, 2, a noise limiter 3 which clips the top of the wave and has a limiting threshold U_0, and a narrow-band output filter 4 with a pass band ΔF and a gain equal to unity (Fig. 2).

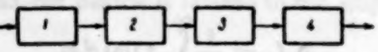

Fig. 2

In performing our computation we shall assume that the pass band Δf of the input unit is such that a) the superposition of pulses beyond the detector can be neglected, and b) the output filter has such a narrow band that the noise at the output of the receiver becomes fluctuation noise. In that case we can assume that the spectral density for the pulses after the detector remains uniform within the limits of the output filter pass band.

The energy spectrum and the parameters of the amplitude distribution for the pulse noise after the detector are determined from formulas (28), (24), and (25); the parameters of the clipped amplitude distribution after the limiter which clips the top of the wave are determined from formulas (15) and (19). In accordance with these formulas we can write the amplitude of the pulse after the detector as

$$y = xk_0\gamma_0, \tag{34}$$

where

$$\gamma_0 = \frac{\Delta f_f}{\Delta f_r}. \tag{35}$$

Under these conditions

$$\overline{\ln y} = \ln a + \ln k_0\gamma_0. \tag{36}$$

The energy spectrum for the pulse noise after the limiter which clips the top of the wave can be expressed in the form of two terms: one for pulses whose amplitude is less than U_0, and the other for clipped pulses in accordance with formula (29):

$$F_0(\omega) = 2n\left\{\frac{1}{\sqrt{2\pi}\sigma}\int_0^{U_0} y\,|G(\omega)|^2 \exp\left[-\frac{(\ln y - \ln ak_0\gamma_0)^2}{2\sigma^2}\right]dy + \right.$$

$$\left. + \frac{1}{\sqrt{2\pi}\sigma}\int_{U_0}^{\infty} \frac{|G_u(\omega, y)|^2}{y} \exp\left[-\frac{(\ln y - \ln ak_0\gamma_0)^2}{2\sigma^2}\right]dy\right\}. \tag{37}$$

Here $G(\omega)$ is the spectral density for a pulse of unit amplitude $y \leq U_0$; $G_u(\omega, y)$ is the spectral density for a clipped pulse $y > U_0$.

Finding the energy spectrum in general form from formula (37) is difficult because the spectral density of the clipped pulses $G_u(\omega, y)$ is a nonlinear function of ω, y and U_0; the shape of this nonlinearity is determined by the shape of the input filter frequency response.

Fig. 3

We shall study the case where the input filter has a frequency response of the single tuned circuit type. Under these conditions the pulse noise after the detector (Fig. 3) consists of a series of exponential pulses with a random amplitude and an identical shape:

$$u(t) = ye^{-\Delta\omega t}, \tag{38}$$

where $\Delta\omega$ is one-half the pass band of the input filter at the 0.7 level.

The spectral density of an exponential pulse with the amplitude y at zero frequency is equal to

$$G(0) = \frac{y}{\Delta\omega}. \tag{39}$$

The spectral density of the portion of the pulse that is clipped from the bottom is equal to

$$G_1(0) = \frac{y - U_0(1+\tau)}{\Delta\omega}, \tag{40}$$

and for a pulse clipped from the top (the shaded portion of Fig. 3) it is

$$G_u(0) = G(0) - G_1(0) = U_0\tau + \frac{U_0}{\Delta\omega}. \tag{41}$$

Substituting the expression

$$\tau = \frac{1}{\Delta\omega} \ln \frac{y}{U_0}, \tag{42}$$

into (41), we obtain

$$G_u(0) = \frac{U_0 \ln \frac{ey}{U_0}}{\Delta\omega}. \tag{43}$$

The expression for the energy spectrum of the pulse noise after the limiter which clips the top of the noise at zero frequency can be written as

$$F_0(0) = \frac{2n}{\Delta\omega^2}\left\{\frac{1}{\sqrt{2\pi}\sigma}\int_0^{U_0} y \exp\left[-\frac{(\ln y - \ln ak_0\gamma_0)^2}{2\sigma^2}\right]dy + \right.$$

$$\left. + \frac{U_0^2}{\sqrt{2\pi}\sigma}\int_{U_0}^{\infty} \frac{\left(\ln\frac{ey}{U_0}\right)^2}{y} \exp\left[-\frac{(\ln y - \ln k_0\gamma_0 a)^2}{2\sigma^2}\right]dy\right\}. \tag{44}$$

Introducing the relative clipping level

$$\psi = \frac{\ln U_0 - \ln ak_0\gamma_0}{\sigma}, \tag{45}$$

we obtain the following result after integrating expression (44):

$$F_0(0) = \frac{2n}{\Delta\omega^2}\left\{D_{\text{м}} + U_0^2 V(\psi)\left[\sigma^2 + (\sigma\psi - 1)^2\right] - \frac{U_0^2 \sigma e^{-\psi^2/2}(\psi\sigma - 2)}{\sqrt{2\pi}}\right\}. \qquad (46)$$

The noise power at the output of the narrow-band filter 4 (Fig. 2) with the frequency response $C(\omega)$ is determined from the formula

$$P_p = F_0(0)\frac{1}{2\pi}\int_{-\infty}^{\infty} C(\omega)\, d\omega = F_0(0)\,\Delta F_e\,. \qquad (47)$$

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THE EFFICIENCY OF INFORMATION TRANSMISSION IN TELEMETERING

I. ANALYSIS WITHOUT CONSIDERING INTERFERENCE IN THE COMMUNICATION CHANNEL

N. V. Pozin

(Moscow)

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The article analyzes the efficiency of the transmission of telemetered information for the basic forms of pulse modulation by the criterion of specific rate of information transmission. Taking account of the equipment errors of signal transformation makes it possible (in part I of the article) to carry out the analysis on the assumption that there is no interference in the communication channel. This corresponds in a number of cases to the practical conditions.

A comparison of the method of transmitting telemetered information is made and is illustrated by data on concrete telemetering systems.

By the efficiency of information transmission we generally mean some complex estimate of the economy or optimality of transmission on the basis of all the factors influencing transmission. These latter, of course, include the effects of interference in the communication channel. In the intial stage, however, it is possible to define and study the efficiency of transmission of telemetered information in the absence of any interference in the communication channel, since even in this case the basic parameters which characterize the transmission are in relations which can be perfected (optimized) according to a chosen criterion of efficiency. In addition, the transmission of telemetered information under conditions where interference plays only a small role is often found in practice.

We call attention to the fact that in what follows equipment errors are taken into account in the analysis of transmission efficiency, which makes it meaningful to carry out such an analysis even in the absence of interference in the communication channel.* The unified approach to the analysis of continuous and discrete methods of transmission is based here on considering the error caused by equipment instability to be identical with the discreteness error.

For the estimation and comparison of the efficiency of information transmission for various methods of telemetering we shall make use of the criterion of "specific transmission rate," which indicates the ratio of the transmission rate R to the bandwidth W occupied in the communication channel. This convenient criterion was proposed in [1].

In the absence of interference and for a uniform distribution of the information transmitted, the transmission rate may be represented by the ratio

$$R = \frac{\log N_A}{T}, \tag{1}$$

where $N_A = 1/\delta_A$ is the precision of the equipment or the number of quantizing levels of the measurement, the reciprocal of the equipment error δ_A, and T is the time necessary for the transmission of one measurement (the telemetering response time). Here and in what follows the symbol "log" will mean logarithm to the base 2.

* By contrast with a number of studies devoted to this question (see, for example, [2]) we have here a distinction between the concepts of "interference" (noise), which arises in a communication channel, and "equipment errors," which are characteristic of the equipment transforming one form of informations carrying signal into another. The necessary presence of equipment error means that even in the absence of transmission interference only a finite number of quantizing levels of the initial information are possible.

The specific rate criterion in the present case can be expressed by

$$\left(\frac{R}{W}\right) = \frac{\log N_A}{TW}. \tag{2}$$

The denominator TW* may be called the frequency inefficiency. The greater TW is for the same response time the more poorly the selected bandwidth is utilized in the communication channel. The frequency inefficiency TW is of interest in a number of cases as an independent criterion, the reduction of which will lead to the perfecting of the telemetering instruments.

We shall derive explicit expressions for the rate R and the specific rate R/W of transmission for the basic forms of modulation used in telemetering.

1. Pulse-Code Modulation (PCM)

Let us consider a coded pulse train with n code pulses. The duration of the pulse train τ consists of the length of the code pulses τ_p and the pulse separation τ_s, as well as the synchronizing pulse. Let us assume that the synchro pulse and the separations have duration equal to $s(\tau_p + \tau_s)$. Then the duration of the pulse train is defined by the formula

$$\tau = (n + S)(\tau_p + \tau_s). \tag{3}$$

We shall express the duration of the front τ_f by means of the required bandwidth W_0 in the communication channel:

$$\tau_f = \frac{1}{W_0}$$

and we shall relate the total duration of the code pulse and separation $\tau_p + \tau_s$ to the duration of the front τ_f by the formula

$$\tau_p + \tau_s = 2k_1\tau_f = \frac{2k_1}{W_0}, \tag{4}$$

where $k_1 \geq 1$. The limiting case $k_1 = 1$ can be represented, for example, as transmission by closely adjoining triangular pulses.

For PCM the transmission time may be considered identical with the duration of the pulse train, that is $T = \tau$. On the basis of (1), (3), and (4) we define the transmission rate by

$$R_{PCM} = \frac{\log N_A}{(n + S)(\tau_p + \tau_s)} = \frac{\log N_A}{2k_1(n + S)} W_0 \tag{5}$$

(for a binary code $\log N_A = n$.) The expression for the specific rate (2) will assume the form

$$\left(\frac{R}{W}\right)_{PCM} = \frac{\log N_A}{2k_1(n + S)} \frac{W_0}{W}. \tag{6}$$

We should remark that the bandwidth W actually represented may coincide with the required bandwidth W_0 or may be greater.

In one channel transmission in most cases S = 2.

In multi-channel transmission the relative weight of the synchro pulse for an individual channel is decreased, and for a large number of channels the synchro pulse may be neglected; then the expression for relative rate for a binary code will have the form

* $TW = 2f_1/2F$ where F is the upper frequency of variation of the parameter measured, and f_1 is the limiting frequency of the low-frequency filter in transmission without secondary modulation.

$$\left(\frac{R}{W}\right)_{PCM} = \frac{1}{2k_1}\frac{W_0}{W}. \tag{6a}$$

2. Pulse-Width and Pulse-Time (Position) Modulation (PWM, PTM)

We shall first consider transmission by pulses with steep fronts. Such transmission is necessary when no special measures are taken in the reception to eliminate the relative instability of the limiting level and the amplitude of the video signal (see below). Let us assume that in this case the duration of the front τ_f is equal to the permissible absolute error of the measurement.

The duration of one cycle is defined by the expression

$$\tau = \tau_d + 2\tau_{min} = (N_A + 2k_2)\,\tau_f = \frac{N_A + 2k_2}{W_0}. \tag{7}$$

Here $\tau_d = N\tau_f$ is the time deviation, $\tau_{min} = k_2\tau_f$ is the pulse (or separation) of minimum duration in PWM (equal to the working pulse in PTM).

The time T in which the readings are established in the telemetering receiver in PWM (or PTM) may be equal to one or several measuring cycle τ, depending on the method of demodulation.

In the general case

$$T = a\tau. \tag{8}$$

We shall call $\underline{a}$ the inertial coefficient.*

The expression for the transmission rate on the basis of (1), (7) and (8) can be represented in the form

$$R_{PWM(PTM)} = \frac{\log N_A}{a\,(N + 2k_2)}\,W_0. \tag{9}$$

By criterion (2) we obtain the formula

$$\left(\frac{R}{W}\right)_{PWM(PTM)} = \frac{\log N_A}{a\,(N_A + 2k_2)}\,\frac{W_0}{W}. \tag{10}$$

In a number ofcases it is possible to carry out narrow-band transmission by PWM (or PTM) that is, transmission by pulses with gradual fronts. Such transmission is more economical by the R/W criterion, but it necessitates some additional complexity in order to avoid errors in the measurement of time intervals as a result of a relative instability of the amplitude of the video signal and the limiting level. For this purpose the receiver uses either automatic regulation of the signal level or limiting of the video pulses by the "tracking" level, that is, a level proportional to the amplitude. In addition, the change to transmission with very gradual front sometimes requires the application of a more stable modulator as a result of the decrease in time deviation (if the same cycle duration is kept). When these conditions are satisfied, the duration of a cycle can be conveniently represented in the following manner:

$$\tau = \tau_d + 2\tau_{min} = N\Delta + 2k_2\tau_f = (Nb + 2k_2)\,\tau_f = \frac{Nb + 2k_2}{W_0}, \tag{11}$$

where Δ is the absolute error of the measurement (by contrast with the previous case, here $\Delta < \tau_f$), $b = \Delta/\tau_f$ is the relative steepness factor of the front.

On the basis of (1), (8) and (11) the expression for the transmission rate can be written in the form

* For example, if the reception is carried out by the average current of the pulses of PWM (PTM is easily transformed with demodulation into PWM) calibrated with respect to amplitude, then a > 1. If the decording of the readings takes place during one cycle, then a = 1.

$$R_{\mathrm{PWM(PTM)}} = \frac{\log N_A}{a\,(N_A b + 2k_2)}\, W_0 \tag{12}$$

and the specific rate criterion will take the form

$$\left(\frac{R}{W}\right)_{\mathrm{PWM(PTM)}} = \frac{\log N_A}{a\,(N_A b + 2k_2)} \cdot \frac{W_0}{W} \,. \tag{13}$$

3. Transmission with Automatic Readjustment

Transmission with automatic readjustment of the response time and the precision [3] is performed with PCM, PWM, and PTM. Let us remember that in those cases when precise telemetering of slow changes of the parameter is required but at the same time a cruder transmission of the rapid changes of the parameter is permissible, it is possible to use automatic readjustment of the transmission. In automatic readjustment an exchange takes place between the response time (duration of the pulse train) and the number of transmitted quantizing levels, depending on the rate of change of information. As a result, the rate of information transmission is decreased. If we compare the ordinary telemetering devices with the readjusting type, we find that in the latter case there is a gain in transmission efficiency, since we require a channel calculated for a lower transmission rate. It is not significant, however, to evaluate this gain by means of the specific rate criterion, since transmission with automatic readjustment is characterized by a lower rate R and a narrower bandwidth W; as a result the rate either does not change (in PCM), or changes by an insignificant amount (in PWM and PTM) by comparison to systems without readjustment.

In connection with this, for comparison of the efficiency of different methods of transmission, including methods with automatic readjustment, we introduce as a uniform criterion for readjusting transmission the concept of "quasi-rate" of transmission. Let us remember that a readjusting device is characterized by a maximum precision N_{max} and a minimum transmission time T_{min}. We shall represent the quasi-rate of readjusting transmission by the expression

$$R_q = \frac{\log N_{max}}{T_{min}} \,.$$

The value of N_{max} and T_{min} coincide with the values of N and T for a non-readjusting system. In other words, in using the concept of "quasi-rate" we make the assumption that the adjust system transmits the same information about the measured parameter as was transmitted earlier by the ordinary non-adjusting system, that is, that the rate of information transmission has not changed. In addition to this, changing to a readjusting system led to a reduction of the bandwidth necessary for transmission on the communication channel. Therefore, the efficiency of readjusting transmission can be evaluated by the ratio of the quasi-rate of transmission to the bandwidth occupied. This makes it possible to compare the efficiency of adjusting and non-adjusting transmission by the uniform criteria R_q/W and R/W, respectively. The criterion R_q/W may be called by analogy the specific quasi-rate of transmission:

$$\frac{R_q}{W} = \frac{\log N_{max}}{T_{min} W} \,. \tag{14}$$

4. Pulse-Frequency Modulation (PFM)*

Let the tracking frequency of pulses in telemetering with PFM change from f_1 to f_2. The lower frequency f_1, in general, must be so chosen that the condition of telemetering response time is satisfied.** However, we must consider the difficulty of producing low-frequency pulses. A very convenient criterion of the degree of realization of a given response time T is the inertial coefficient which in the present case is equal to

* The material of this section is equally applicable to frequency modulation as well, if the latter refers to "low-frequency telemetering," that is, a low-frequency change with large relative deviation. However, in practice low-frequency systems are usually pulse-frequency systems, since for low-frequency it is easier to form pulses than sinusoidal oscillations.

** For example, if the reception of pulse-frequency telemetering is effected by the average current of short pulses calibrated with respect to amplitude and duration (as in the widely used capacitor-type frequency meter), then the frequency f_1 must be matched with the receiving equipment so as to exclude pulsations in the readings of the instrument.

$$a = T f_1. \tag{15}$$

Furthermore, the upper frequency f_2 is defined by considerations of the precision of telemetering. It is obvious that the precision may be defined by the formula

$$N_A = \frac{f_2 - f_1}{\Delta}, \tag{16}$$

where Δ is the absolute equipment error.

After some simple transformation, we obtain from (16)

$$f_1 = \frac{f_2}{N_A \frac{\Delta}{f_2} + 1}.$$

Using this relation and introducing the notation $c = \Delta/f_1$, we can express T from (15)

$$T = \frac{a}{f_2}(N_A c + 1).$$

According to (1), the transmission rate is equal to

$$R_{\mathrm{PFM}} = \frac{\log N_A}{a(N_A c + 1)} f_2. \tag{17}$$

We note that $N_A c + 1 = f_2/f_1$. Obviously the upper frequency f_2 defines the necessary bandwidth W_0 in the communication channel. For narrow-band telemetering in a communication channel with secondary modulation it is sufficient to transmit two side frequencies, that is, $W_0 = 2f_2$. Then

$$\left(\frac{R}{W}\right)_{\mathrm{PFM}} = \frac{\log N_A}{2a(N_A c + 1)} \frac{W_0}{W}. \tag{18}$$

5. Analysis and Comparison of the Efficiency of Telemetering Transmission Methods

The resulting expressions R/W are, as can be seen, functions of a number of arguments. An analysis of the efficiency can conveniently be made by means of graphs, choosing in the present case, that is, in the absence of

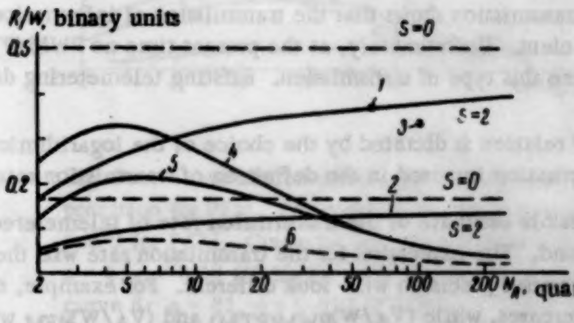

Fig. 1. Transmission efficiency as a function of telemetering precision for PCM and PWM (PTM). The PWM (PTM) transmission is of the wide-band type, that is, it uses pulses with steep fronts for b = 1.

interference in the communication channel, the equipment precision N_A of the transmission as an argument. For simplicity of analysis we shall consider $W_0/W = 1$.

In Fig. 1, using formula (6), we have constructed for a binary code (log N = n) the curves of $(R/W)_{\mathrm{PCM}}$ as a function of N_A for S = 2, with $k_1 = 1$ for curve 1 and $k_1 = 3$ for curve 2. As the precision N_A increases, the efficiency

of coded transmission increases, since the "weight" of the sync. pulse, which does not carry useful information in the coded pulse train is decreased. These curves approach the dashed straight lines (S = 0) constructed according to formula (6a). From a comparison of curves 1 and 2 it can be seen that an increase in k_1, that is, an increase in pulse duration as compared to the duration of the fronts, decreases the transmission efficiency, since in this case there is an increase either in the time T (for W_0 = constant), or the necessary bandwidth W_0 (for T = constant).

We shall indicate on this graph the values of R/W for concrete telemetering devices. In the Appendix we list the data for a number of devices; it follows from these data that the efficiency of the coded devices developed by the VNIIE(TK-2) [4] and the TsNIIKA [5], are defined respectively by points 1 and 3 in Fig. 1.

We now pass to the case of PWM (PTM). The $(R/W)_{PWM(PTM)}$ curves calculated by formula (10), characterizing transmission with fairly steep fronts (b = 1), are also constructed in Fig. 1: curve 4 for k_2 = 1 and curve 5 for k_2 = 3 (in both cases a = 1). These curves have a maximum whose physical meaning is the following: At the beginning, for small N_A for the duration of the entire pulse train great specific weight is attached to the minimum pulse (and separation), which does not in itself carry any useful information. The increase of N_A at first increases the efficiency, since there is an increase in the time deviation τ_d (transmitting useful information) by comparison with the minimum pulse, but after this the transmission efficiency decreases, tending to zero, since the duration of the entire pulse train for W_0 = constant (or the bandwidth W_0 for T = constant) increases faster than the number of binary units transmitted by the pulse train.* We note that the difference in the values of k_2 is pronounced only in the case of small N_A: for $N_B \geq 50$, curves 4 and 5 become practically identical. It is obvious that an increase in the inertial coefficient (curve 6 for k_2 = 3 and a = 3) sharply reduces the transmission efficiency.

Examination of the graphs in Fig. 1 shows that the transmission of information by PCM for k_1 = 1 is always more efficient than wide-band transmission (that is, transmission by pulses with steep fronts) using PWM (PTM). However, for $k_1 \geq 3$, coded transmission becomes more efficient than wide-band transmission with PWM (PTM) only for large values of N_A.

Figure 2 shows graphs constructed by formula (13) for narrow-band transmission by PWM(PTM) for several values of the relative steepness factor b of the fronts: from b = 1 to b = 1/20 (the solid lines refer to a = 1, the dashed lines to a = 3; k_2 = 3 in all cases). For comparison we also show here curves 1 and 2 for PCM for the values k_1 = 1 and k_1 = 3, respectively.

It is clear from the graphs of Fig. 2 that transmission by PWM (PTM) with b ≥ 1/2 is more efficient than transmission by PCM for k_1 = 3 (curve 2), while PWM (PTM) with b = 1/20 is more efficient even than PCM for k_1 = 1 (curve 1) for a wide range of values of N_A. We note that the decrease of b not only increases the efficiency but also displaces the maximum of the efficiency into the region of greater N_A values.

Thus, comparison with coded transmission shows that the transmission of information by PWM (PTM) using pulses with gradual fronts is very efficient. Unfortunately, at the present time no PWM (PTM) telemetering devices have yet been developed which realize this type of transmission. Existing telemetering devices, for example, those

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* Of course, such a type of functional relation is dictated by the choice of the logarithmic—in the present case, binary—units of measurement of information imposed in the definition of transmission rate R.

Besides this there is another possible estimate of the transmission rate of telemetered data—by the number of quantizing levels transmitted per second. The expression for the transmission rate will then have the form $V_A = N_A/T$, and the graphs of specific rate V_A/W versus precision will look different. For example, the graph of $(V_A/W)_{PCM}$ will increase monotonically as N_A increases, while $(V_A/W)_{PWM(PTM)}$ and $(V_A/W)_{PFM}$ will tend to a finite limit different from zero.

In a number of cases, especially in the analysis of transmission efficiency where interference is taken into consideration, it is necessary that the changes in precision N be reflected more clearly in the graphs of the efficiency criterion as a function of such transmission parameters as the bandwidth in the channel, interference level, etc. Under these conditions an estimate by the criterion V/W = N/TW may be found to be preferable to the R/W = (log N)/TW estimate in which the logarithm in the numerator smooths the effect of changes in the precision N (under the influence of interference the transmission precision N is determined both by the equipment error and by the error caused by interference in the communication channel).

of Bristol or the VST-1 (IAT AN SSSR Institute of Automation and Remote Control of the Academy of Sciences of the USSR) [6], belong to the class of wide-band devices; their efficiency is defined respectively by points 3 and 4 in Fig. 2 (the data for these devices are given in the Appendix).

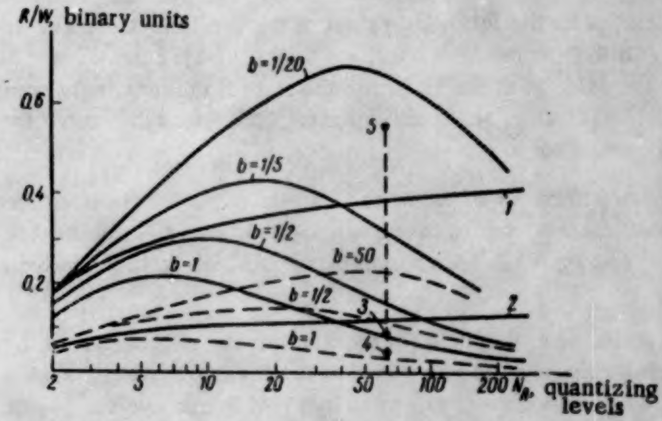

Fig. 2. Transmission efficiency as a function of telemetering precision for narrow-band PWM-PTM. Curves 1 and 2 for PCM are shown for comparison. The solid curves refer to a = 1, the dashed curves to a = 3.

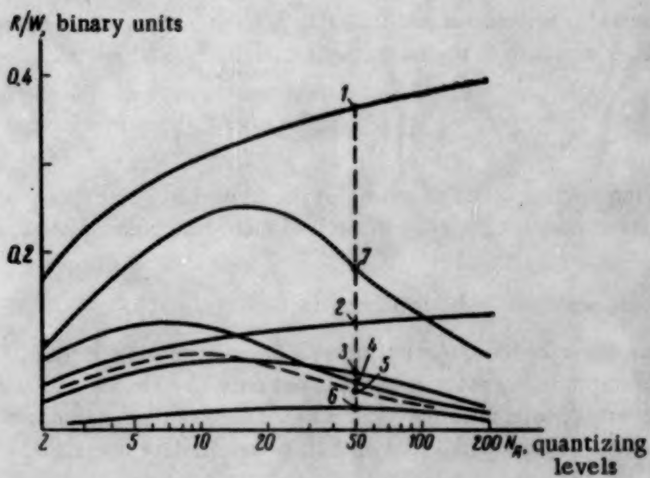

Fig. 3. Transmission efficiency as a function of telemetering precision for PFM. For comparison the figure shows curves 1 and 2 for PCM. Curve 3: a = 15, c = 0.04 (ChTI-1); curve 4: a = 5, c = 0.18 (ChIS-1); curve 5: a= 12, c = 0.08 (ChI-1); curve 6: a = 81, c = 0.012 (TNCh).

We shall show for comparison the data on the efficiency of PWM with automatic readjustment (PWM-AR). A breadboard model of such a device was made at the (Institute of Automation and Remote Control of the Academy of Sciences of the USSR); the data for this breadboard model given in the Appendix. The PWM-AR device transmits pulses with fairly steep fronts ($\tau_f = \Delta$, that is, b = 1), but by virtue of its automatic readjustment it has a fairly high transmission efficiency. Characterizing the transmission efficiency of PWM-AR by the specific "quasi-rate" of (14), we obtain point 5 in Fig. 2.

We shall now analyze PFM. Formula (18) is analogous in structure to formula (13), and therefore in the case of PFM we should expect curves similar to those shown in Fig. 2. Since PFM devices have become widespread in practice and a number of concrete developments are known today, it is interesting to construct the efficiency curves

for coefficients a and c whose values are close to the real values. In the Appendix we give the data which make it possible to determine these coefficients for four telemetering devices: ChTI-1 (IAT AN SSSR) [7], ChIS-1 (TsLEM) [8], ChI-1 (TsNIIKA) [9], TNCh (TsNIEL–Elektropult) [10]. In these data it is assumed that all the devices have the same precision $N_A = 50$ (which is close to the real value) and occupy channels with frequency bands of the minimum width necessary for passing the first harmonic for an upper frequency of f_2* (that is $W_0/W = 1$).

Figure 3 shows the curves for the following values of the coefficients: curve 3: a = 15, c = 0.04 (ChTI-1); curve 4: a = 5, c = 0.18 (ChIS-1); curve 5: a = 12, c = 0.08 (ChI-1); curve 6: a = 81, c = 0.012 (TNCh). It is easy to observe that the lower the value of c, the farther to the right is the maximum point of the curve. All four curves, for the values of N_A useful in practice, are much lower than the "standard" curve for coded transmission for $k_1 = 3$ (curve 2), not to mention curve 1 for $k_1 = 1$.

The comparatively low efficiency of pulse-frequency transmission is determined chiefly by the comparatively large values of inertial coefficient a. Let us take as an example curve 4, constructed for the coefficients of the ChTI-1. If we replace a = 15 by a = 5,** we obtain curve 7, which up to the value $N_A = 100$ is higher than the "standard" curve 2.

Thus, the lower the value of a, the higher is the transmission efficiency. In the limit a = 1, that is, the most efficient reception of readings would be in one period of the lower frequency. Such a reception method is possible in principle, but it is obviously analogous to reception by PWM in one cycle,*** that is, at least with respect to the method of reception, PFM would reduce to PWM.

In addition, transmission efficiency increases as c decreases, or which is the same thing for fixed precision N_A as the factor $Nc + 1 = f_2/f_1$ in the denominator of (17) decreases. Clearly, a decrease in c or in f_2/f_1 (for a given value of f_1) is equivalent to a decrease in the frequency deviation or the absolute transformation error Δ.

A decrease in the inertial coefficient a and the ratio f_2/f_1 is nothing but a decrease in the components of the denominator in (17), that is, a decrease in the frequency inefficiency TW, since

$$2a\,(N_A c + 1)\left(\frac{W}{W_0}\right) = 2a\,\frac{f_2}{f_1}\left(\frac{W}{W_0}\right) = TW.$$

In the four telemetering devices taken as examples the transmission precision is the same ($N_A = 50$), and therefore the comparative transmission efficiency of each of them (see points 3, 4, 5, and 6 in Fig. 3) is determined by the values of TW.

Summarizing the above, we observe the following facts:

1. The resulting expressions for the R/W criterion as a function of all the fundamental electrical parameters characterizing telemetering transmission make it possible not only to estimate and compare with great clarity the transmission efficiency of concrete telemetering systems based on different methods of modulation but also to show ways of perfecting these systems and the conditions of their most efficient operation.

2. A comparison of the telemetering transmission methods PCM, PWM (PTM) and PFM (in a channel without interference) by the R/W criterion indicates, in particular, the following:

a) As telemetering precision increases, so does the efficiency of coded transmission, but for precision values of the order of $N_A = 100$ and less, PCM may be less efficient than other forms of modulation;

* This condition is not always realized in practice, especially in industrial telemetering. However, the given analysis is of value only when the question of transmission efficiency is of interest and is isolated from such factors lowering the efficiency as excess bandwidth of the filters in the communication channel.

** This means for the ChTI-1, for example, a change from an inertial output device with T = 3 seconds to a device T = 1 second, provided, of course, that the receiver of the information–a man or an automatic device–is in a position to provide such a response time.

*** For example, if PWM reception in one cycle is affected by a transformation of pulse duration into amplitude by means of linear scanning in the receiver, it is possible to make a similar transformation of the duration of the period (or a part of the period) of the oscillations with PFM into amplitude by means of hyperbolic scanning, which in a number of cases is well approximated by exponential scanning.

b) For these precision values ($N_A < 100$), narrow-band telemetering (that is, telemetering based on the transmission of pulses with gradual fronts by PWM (PTM) makes it possible to attain the highest transmission efficiency.

APPENDIX

Technical Data for Several Telemetering Devices

Name of organization and initials of device	T, sec	n	$\frac{W_0}{2}$, gc	S	N_A	$\tau_p + \tau_s = \frac{T}{n+S}$, sec	$K_1 = \frac{W_0}{2}(\tau_p + \tau_s)$	TW ($W = W_0$)	$\frac{R}{W} = \frac{\log N_A}{TW}$ ($W = W_0$)
VNIIE; TK-2	0.266	6	30	2	64	0.033	1	16	0.375
TsNIIKA*	0.25	8	40	2	100	0.025	1	20	0.33

* In the TsNIIKA system a binary decimal code is used; out of n = 8 code pulses, 4 transmit of the tens in binary code, and the other 4 transmit the units. Thus, here, N = 100, while $\log N \neq n$.

Name of organization and initials of device	τ, sec	τ_d sec	$\frac{W_0}{2}$, sec	T, sec	N_A	$a = \frac{T}{\tau}$	$\Delta = \frac{\tau_d}{N_A}$, sec	$V = \Delta W_0$	TW ($W = W_0$)	$\frac{R}{W} = \frac{\log N_A}{TW}$ ($W = W_0$)
Bristol	16	14	1.8	16	50	1	0.28	1	56	0.1
IAT AN SSSR; VST-1	2	1.6	15	4	50	2	0.032	1	120	0.047

Name of organization and initials of device	f_1, gc	f_2, gc	$\frac{f_2}{f_1}$	T, sec	N_A	$a = Tf_1$	$\Delta = \frac{f_2 - f_1}{N_A}$, gc	$c = \frac{\Delta}{f_1}$,	$TW = 2a\frac{f_2}{f_1}$ ($W = W_0$)	$\frac{R}{W} = \frac{\log N_A}{TW}$ ($W = W_0$)
IAT AN SSSR ChTI-1	5	15	3	3	50	15	0.2	0.04	90	0.062
TsLEM; ChIS-1	1	10	10	5	50	5	0.18	0.18	100	0.056
TsNIIKA*; ChI-1	4	20	5	3	50	12	0.32	0.08	120	0.047
TsNIEL-Élektropul't TNCh**	27	44	1.63	3	50	81	0.34	0.012	260	0.02

* See the remark* on page 1081.

Name of organization and initials of device	T_{max} sec	T_{min} sec	$N_{A\,max}$	$N_{A\,min}$	$\tau_f = \frac{T_{max}}{N_{max}}$, sec	$W_0 = \frac{1}{\tau_f}$, gc	$\frac{R}{W} = \frac{\log N_{max}}{T_{min} W}$
IAT AN SSSR PWM-AR	10	2	50	10	0.2	5	0.56

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All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. ***Some or all of this periodical literature may well be available in English translation.*** **A complete list of the cover-to-cover English translations appears at the back of this issue.**

THE STARTING-UP TIME AND ITS EFFECT ON THE CHARACTERISTICS OF GYRO MOTORS WITH HYSTERESIS

N. Z. Mastyaev and I. N. Orlov

(Moscow)

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An analytical expression for determining the starting-up time of hysteresis gyro motors, which yields results that are in good agreement with experimental data, is derived. The maximum calculated power of a hysteresis gyro motor which would secure the assigned starting-up time is derived. An analysis of the starting-up time effect on the characteristics of hysteresis gyro motors, e.g., power consumption, efficiency, cos φ, and heating, is presented.

Electrically driven gyroscope gained widespread use as position data transmitters and as differentiating and integrating elements of automatic systems.

The gyroscope starting-up time often determines the time when the entire device is ready for operation, and, therefore, its magnitude is in many cases critical in estimating gyro motors. In connection with this, the possibility of calculating the starting-up time with sufficient accuracy in designing gyro motors is an important practical problem.

In designing gyro motors, it is even more important to determine the maximum calculated motor power which would secure the assigned starting-up time.

The present article presents the solution of the stated problem with respect to synchronous hysteresis gyro motors, which are recently being widely used for driving gyroscope rotors in highly accurate systems. At the same time, this article presents a general analysis of the starting-up time effect on the characteristics of hysteresis gyro motors, e.g., power consumption, efficiency, cos φ, and heating.

1. Mechanical Characteristic and the Starting-Up Time of Hysteresis Gyro Motors

The mechanical characteristic $\omega = f(M)$ of the majority of actual hysteresis gyro motors has the shape of the curve a in Fig. 1. The maximum electromagnetic moment of a hysteresis motor usually acts at the starting ($\omega = 0$, $s = 1$). Even in the case of motors where the effect of eddy currents in the rotor material is negligibly small, the maximum synchronous moment M_{ms} is smaller than the starting (short-circuit) torque M_{sc}.

For well-designed hysteresis motors, the mechanical characteristic coefficient c_m, which is equal to the ratio of the starting torque M_{sc} to the maximum synchronous moment M_{ms}, usually lies within 1.2 to 1.8.

The "useful" load of a gyro motor is the moment M_d of mechanical losses—losses in its own bearings and of losses in friction between the rotor and the surrounding medium. The gyro motor's operating point will be the point of intersection between the motor's mechanical characteristic $\omega = f(M_{mo})$ and the drag (mechanical loss) torque curve $M_d = f(\omega)$ (Fig. 1).

The $k_m = M_{sc}/M_{2n}$ ratio, where M_{2n} is the rated load torque, is called the starting torque factor; it determines the maximum motor torque for a certain given rated torque. The motor's overload capacity factor $k_0 = M_{ms}/M_{2n}$, in turn, determines the degree of loading and the motor's overload capacity under synchronous conditions.

The mechanical characteristic coefficient c_m, the overload capacity factor k_0, and the starting torque factor $k_m = c_m k_0$ are the most important indices of the mechanical characteristic of a hysteresis gyro motor and they determine its starting-up time t_s, all other conditions being equal.

The starting-up process is described by the equation

$$M_{\mathrm{mo}}(\omega) - M_{\mathrm{d}}(\omega) = J\frac{d\omega}{dt}, \tag{1}$$

where $M_{\mathrm{mo}}(\omega)$ is the motor's torque in dependence on angular velocity (g-cm), $M_{\mathrm{d}}(\omega)$ is the load (drag) torque of the motor (g-cm), which is a function of the angular velocity, J is the moment of inertia of the motor's rotating parts (g-cm sec^2), and ω is the angular velocity of the motor (rad/sec).

As is known, the area enclosed by the mechanical characteristic and the drag torque curve of a gyro motor (the shaded area in Fig. 1) is used as the measure of the starting-up time t_s for certain given J and ω_{d} values.

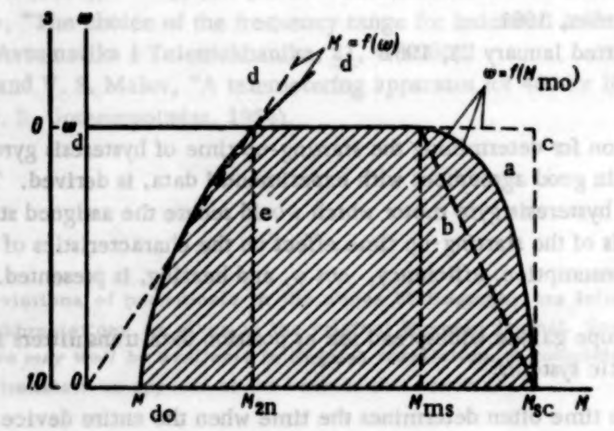

Fig. 1. Mechanical characteristics $\omega = f(M_{\mathrm{mo}})$ and the drag torque curves $M_{\mathrm{d}} = f(\omega)$ of a hysteresis gyro motor.

The gyro motor's starting-up time can be determined by solving Eq. (1) with respect to time $\underline{t}$:

$$t_{\mathrm{p}} = \int_{\omega=0}^{\omega_{\mathrm{d}}} \frac{J}{M_{\mathrm{mo}}(\omega) - M_{\mathrm{d}}(\omega)}\, d\omega. \tag{2}$$

By expressing the mechanical characteristic $M_{\mathrm{mo}}(\omega)$ and the load curve $M_{\mathrm{d}}(\omega)$ analytically, we obtain an expression for the starting-up time t_s of the motor for certain given J, ω_{d}, $M_{2\mathrm{n}}$, k_m, and k_o values.

The configuration of the actual mechanical characteristic of a hysteresis motor makes it possible to replace it by two straight-line sections (curve $\underline{b}$ in Fig. 1) and to approximate it by the equations:

$$M_{\mathrm{mo}}(\omega) = \begin{cases} M = M_{2\mathrm{n}}\left[k_{\mathrm{m}} - (k_{\mathrm{m}} - k_{\mathrm{o}})\dfrac{\omega}{\omega_{\mathrm{d}}}\right] & \text{for } 0 \leqslant \omega \leqslant \omega_{\mathrm{d}}, \\ 0 \leqslant M \leqslant k_{\mathrm{o}} M_{2\mathrm{n}} & \text{for } \omega = \omega_{\mathrm{d}} \end{cases} \tag{3}$$

The drag torque curve $M_{\mathrm{d}} = f(\omega)$ is generally a complex function of the angular velocity, and it has a constant component M_{do}, which is independent of the angular velocity and which is determined by static friction in the bearings as well as a component which is approximately proportional to $\omega^{1.5}$ and which characterizes friction against the surrounding medium. However, for the sake of simplicity, we shall represent the drag torque curve in the shape of the straight line $\underline{d}$ (Fig. 1) by using the equation

$$M_{\mathrm{d}}(\omega) = M_{2\mathrm{n}}\frac{\omega}{\omega_{\mathrm{d}}} \text{ for } 0 \leqslant \omega \leqslant \omega_{\mathrm{d}}. \tag{4}$$

Comparison between the Theoretical and Experimental Data Used for Determining the Starting-up Time of Hysteresis Gyro Motors

Quantities	Meas. units	Gyro motor specimens						
		1	2	3	4	5	6	7
K	g-cm sec	27	28	49.5	63	230	—	94000
M_{2n}	g-cm	3.0	2.6	4.3	3.0	3.4	14	110
k_o	–	1.7	2.2	1.65	2.4	2.44	2.08	1.1
c_m	—	1.6	1.5	1.2	1.53	1.57	1.2	1.75
k_m	—	2.7	3.3	2.0	3.67	3.83	2.5	1.9
t_s,(experiment)	sec	5.0	4.9	9.0	7.0	26.9	60	1080
t_l [calc. by means of (6)]	sec	6.1	5.17	9.6	8.8	27.5	76	1390
t_s [calc. by means of (12)]	sec	6.1	5.1	10	8.7	27.2	74	1210
t''_s/t_s	—	1.22	1.05	1.11	1.25	1.04	1.23	1.12

It is readily seen that, in approximating $M_{mo}(\omega)$ and $M_d(\omega)$ by Eqs. (3) and (4), one can expect higher theoretical values for the starting-up time t_s in comparison with experimental data for the majority of gyro motors.

By using Eqs. (2)-(4), we obtain:

$$t_s = \frac{J}{M_{2n}} \int_{\omega=0}^{\omega_d} \frac{d\omega}{k_m - (k_m - k_o + 1)\frac{\omega}{\omega_d}}. \tag{5}$$

Integration yields the equation

$$t_s = \frac{K}{M_{2n}} \frac{2,3}{k_m - k_o + 1} \lg \frac{k_m}{k_o - 1}, \tag{6}$$

where $K = J\omega_d$(g-cm sec) is the gyroscope's angular momentum, and t_s is given in seconds.

The obtained expression (6) makes it possible to determine the starting-up time t_s for a given hysteresis gyro motor with known K, M_{2n}, k_o, and k_m values, and also to analyze the effect of various gyro motor parameters on the starting-up time.

The table provides experimental data for a number of hysteresis gyro motor specimens and the starting-up time t_s values, which were calculated by using Eq. (6). The discrepancy between theoretical and experimental data does not exceed 25%. Although this cannot be considered as a sufficiently good agreement, it is entirely acceptable for approximate estimates. Here, one should mention the difficulties encountered in determining the mechanical characteristic of gyro motors, especially with respect to the maximum synchronous moment M_{ms}, which additionally reduces the accuracy of calculations.

For equal K and M_{2n} values, the starting-up time is the shorter, the larger the starting torque factor k_m, while, for a given k_m, it is the shorter, the larger the motor's overload capacity factor k_o (or the smaller the c_m coefficient). The dependence of $t_s M_{2n}/K$ on k_m for different c_m values, which was calculated by using (6) and is shown in Fig. 2, confirms the above statement. The minimum starting-up time t_s for a given k_m will be obtained for $k_o = k_m$ ($c_m = 1$) (curve b in Fig. 1).

According to (6), for certain given k_o and k_m, the starting-up time t_s increases with an increase in the angular momentum K. This increment is the larger, the smaller the gyro motor's rated torque M_{2n}, since this signifies a decrease in the excess moment $M_{mo} - M_d = M_{2n}[k_m - (k_m - k_o + 1)\frac{\omega}{\omega_d}]$. By expressing K and M_{2n} in terms of the dimensions and parameters of the gyro motor [1], their effect on the starting-up time can readily be determined.

Actually, the expression for the angular momentum for a cylindrical rotor (Fig. 3) is given by

$$K = J\omega_d = 1.047 \cdot 10^{-8} \gamma D_o^5 \frac{L_o}{D_o}\left[1 - \left(\frac{D_1}{D_o}\right)^4\right] n \text{ (g-cm sec)}. \quad (7)$$

while the following equation can be used for calculating the drag torque in a certain given range of D_o and n:

$$M_{2n} = \frac{P_{2n} \cdot 10^5}{1.03n} = 0.97\,(a_0\beta_0 \sqrt{\rho\mu'})\sqrt{n^3}\,D_o^4\left(1 + 5\frac{L_o}{D_o}\right)10^5 \text{ (g-cm)} \quad (8)$$

Here, D_o, D_1, and L_o are the dimensions given in Fig. 3 in centimeters, γ is the specific weight of the rotor material in g/cm^3, n is the gyro motor's angular velocity in rpm, β_0 is the ventilation loss factor, ρ is the working medium density in g sec^2/cm^4, μ' is the viscosity coefficient of the working medium, $a_0 = 1 + (\Delta P_{bear}/\Delta P_{vent})$ is a coefficient which takes into account the portion of losses in the bearings in the over-all drag torque, ΔP_{bear} is the friction loss in the bearings in watts, and ΔP_{vent} is the loss resulting from friction in the medium (ventilation loss) in watts.

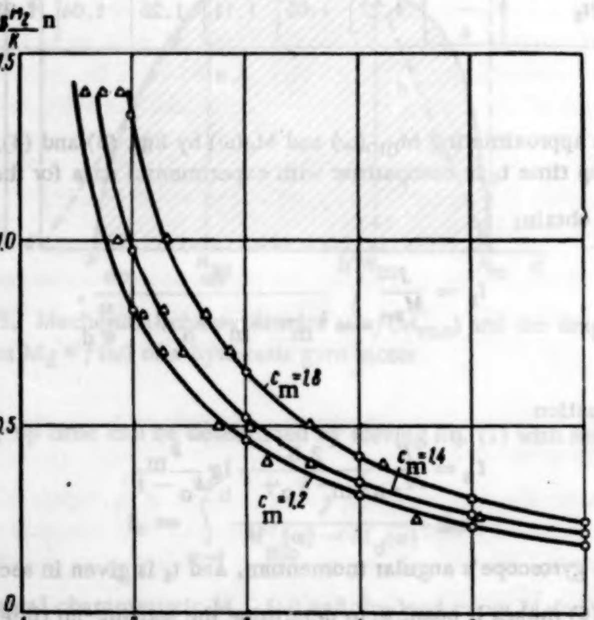

Fig. 2. Relative starting-up time $t_s M_{2n}/K$ of a hysteresis gyro motor in dependence on k_m for different c_m values. The points denote the results of calculation according to Eq. (6), and the triangle denotes the results obtained by using Eq. (12).

By means of (7) and (8), we readily obtain:

$$\frac{K}{M_{2n}} = 1.08 \cdot 10^{-10} D_o \frac{L_o / D_o}{1 + 5L_o / D_o} \frac{\gamma}{\sqrt{n}} \frac{1 - (D_1 / D_o)^4}{(a_0\beta_0 \sqrt{\rho\mu'})} \text{ (sec.)}. \quad (9)$$

As follows from expressions (6) and (9), for certain given k_o and k_m, the starting-up time is the shorter, the smaller the gyroscope dimension D_o, the greater the angular velocity n, the larger the ratio D_1/D_o of the rotor diameters, the smaller the L_o/D_o ratio and the specific weight of the rotor material, and the greater the density of the medium.

The conclusions drawn here apply best in the case of gyroscopes with small bearing losses ($a_0 \approx 1$), when the dependence of the drag torque on angular velocity and the gyroscope dimensions can be represented by Eq. (8) with

sufficient justification. If the bearing loss is considerable ($a_0 > 1$), the drag torque is largely determined by the rotor's weight, which materially changes the conclusions following from expression (9). The latter applies especially to gyroscopes with a small angular momentum and to vacuum gyroscopes.

2. Theoretical Power of Hysteresis Gyro Motors

In designing gyro motors, a problem which is opposite to that considered above must be solved: We must find the power necessary for securing the assigned starting-up time t_s, which is to be used as the basis in designing the motor. The practical problem can be reduced, in the first place, to the determination of the value of coefficient k_m (or k_o) that is necessary for securing the required starting-up time t_s for certain given K and M_{2n} values and for a given mechanical characteristic coefficient (c_m = 1.2-1.8). The value of k_o can be determined by graphically solving Eq. (6), which can readily be transformed in to the following form:

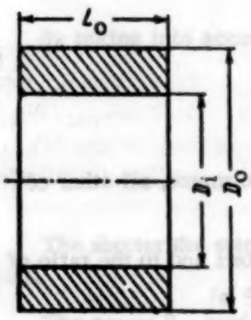

Fig. 3

$$\frac{1}{2.3}\,\frac{t_s M_{2n}}{K}\left[1 + (c_m - 1)\,k_o\right] = \lg c_m \frac{k_o}{k_o - 1}\,. \tag{10}$$

However, the use of this expression is difficult, especially for general analysis purposes, and it is desirable to derive a simpler relationship between k_m and the motor's starting-up time t_s. It would be more correct to eliminate the logarithmic dependence (10) by using expansion in a power series. However, for actual values of the starting torque factor (k_m = 1.5-5.0), the series must contain not less than two or three terms, which results in a rather complex equation for k_m. It proved to be simpler to use the approximation of the $t_s M_{2n} / K = f(k_m, c_m)$ dependence in the region of actual values of c_m (c_m = 1.2-1.8) and k_m (k_m = 1.5-5.0) which is shown in Fig. 2. The approximating equation is given by

$$k_m = 1 + \frac{0.725 c_m}{\frac{t_s M_{2n}}{K}}\,. \tag{11}$$

In this case, the starting-up time t_s is determined by the equation

$$t_s = \frac{K}{M_{2n}}\,\frac{0.725 c_m}{k_m - 1}\,. \tag{12}$$

Expression (12) was used for calculating the starting-up time t_s^* values given in the table and in Fig. 2. The agreement between these values and the results of calculations where (6) is used justifies the use of Eqs. (11) and (12) for practical calculations and general analysis.

Hysteresis motors use the maximum electromagnetic power during the starting (short-circuit) period. If the electromagnetic power under the rated conditions is equal to

$$P_{er} = \frac{P_{2n}}{\eta_2} = \frac{1.03 n M_{2n} \cdot 10^{-5}}{\eta_2}\,, \tag{13}$$

the electromagnetic power at the starting point will be found by using the equation

$$P_{ew} = k_m \frac{P_{2n}}{\eta_2} = \frac{1.03 n M_{2n} k_m \cdot 10^{-5}}{\eta_2}\ (\text{w}), \tag{14}$$

where η_2 is the "output" efficiency, which determines the portion of surface losses ΔP_{sur} in the material of the rotor that are due to higher harmonics:

$$\eta_2 = \frac{P_{2n}}{P_{er}} = 1 - \frac{\Delta P_{sur}}{P_{er}}\,, \tag{15}$$

The maximum electromagnetic power P_{ew} represents the calculated power of a hysteresis motor. It should be emphasized that precisely this point in the starting of hysteresis motors can be most accurately calculated by using the energy method. By using (8), we can readily find M_{2n} with respect to the assigned angular momentum K, the starting-up time t_s, and the given rotor dimensions, while the starting torque factor k_m can be found by using (11) with respect to the assigned mechanical characteristic coefficient c_m, after which the power P_{ew} (14) for which the gyro motor is designed can readily be determined.

By using Eqs. (11) and (14), the calculated power P_{ew} of a hysteresis gyro motor can readily be expressed directly in terms of the starting-up time t_s:

$$P_{ew} = 1{,}03 \cdot 10^{-5}\, M_{2n} \frac{n}{\eta_2}\left(1 + \frac{0.725 c_m}{t_s \frac{M_{2n}}{K}}\right) (\mathrm{w}). \tag{16}$$

The shorter the starting-up time t_s, the larger the calculated power P_{ew} of the gyro motor, all other conditions being equal.

By using (7), (8), and (16), the calculated power P_{ew} can be related to the parameters and to the ratio of the gyro motor dimensions for a certain given angular momentum K and the starting-up time t_s:

$$P_{ew} = 0{,}956 \cdot 10^{5} K \frac{(a_0\beta_0 \sqrt{\rho\mu'})}{\gamma \frac{L_o}{D_o}} \frac{\sqrt{n^3}}{\eta_2 D_o} \frac{1 + 5\frac{L_o}{D_o}}{\left[1 - \left(\frac{D_i}{D_o}\right)^4\right]} \times$$

$$\times \left\{1 + \frac{0{,}78 \cdot 10^{-10}}{(a_0\beta_0 \sqrt{\rho\mu'})} \frac{c_m}{t_s \sqrt{n}} \frac{D_o \frac{L_o}{D_o}\gamma}{\left(1 + 5\frac{L_o}{D_o}\right)} \left[1 - \left(\frac{D_i}{D_n}\right)^4\right]\right\} (\mathrm{w}). \tag{17}$$

The shorter the starting-up time t_s, the larger the second term in the large brackets in expression (17) (the larger the necessary excess moment of the motor), and the smaller the effect of the surrounding medium parameters ($\beta_0 \sqrt{\rho\mu'}$), the dimensions $(D_o, L_o/D_o,\ D_i/D_o)$ and the specific weight of the gyroscope rotor on the calculated power, since the drag torque portion in the calculated torque of the gyro motor decreases. The remarks made in the analysis of expression (9) also apply to the above conclusions.

3. Effect of the Starting-up Time t_s on the Characteristics of Hysteresis Gyro Motors

The found relationship between the calculated power P_{ew} of hysteresis gyro motors and the starting-up time t_s makes it possible to design gyro motors with an assigned t_s. Since the calculated power here depends on the starting-up time, the latter also determines to a large extent the electrical gyro motor characteristics (the current I_1, the power consumption P_1, $\cos\varphi$, and the efficiency η) and its heating.

The effect of the starting-up time t_s on the gyro motor characteristics can be conveniently determined by comparing the characteristics of an actual motor with a given k_m (that secures the assigned t_s value) with an idealized motor which has no torque margin ($k_o = k_m = 1$). The mechanical characteristic of such a motor is given by the curve e in Fig. 1. In this case, as can readily be seen from (6), the starting-up time is $t_s = \infty$ (the angular velocity rise follows an exponential curve). Comparison with such an idealized motor makes it possible qualitatively to estimate the effect of any finite value $t_s \neq \infty$ on the hydraulic motor's characteristics.*

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* We assume that both motors to be compared have equal angular momentums K, equal absolute dimensions and their ratios (which are determined by the value of K), equal stator winding specifications, and equal values of the hysteresis angle $\sin\gamma_0$ in the magnetic materials of the rotor's active part, while it is assumed that these materials are used in the optimum manner. The latter means that the efficiency η and $\cos\varphi$ values are equal at the calculated starting point. The motors to be compared differ with respect to the power at the calculated point ($k_m \frac{P_{2n}}{\eta_2}$ and $\frac{P_{2n}}{\eta_2}$), the air-gap induction B_δ, the "hardness" of the rotor's magnetic materials, and the operating voltages U_1 acting in the stator windings.

By using the fact that, for an optimally designed gyro motor, the required over-all magnetizing force F_1 in the stator winding is determined by the air-gap induction value B_δ, which is proportional to $\sqrt{P_{ew}}$, all other conditions being equal, we readily obtain:*

$$\frac{I_1}{I_{10}} = \frac{F_1}{F_{10}} = \frac{B_\delta}{B_{\delta_0}} = \frac{\sqrt{P_{ew}}}{\sqrt{(P_{ew})_0}} = \sqrt{k_m} \tag{18}$$

By taking into account (11), we obtain the relationship for the stator's phase current:

$$I_1 = I_{10}\sqrt{1 + \frac{0.725 c_m}{\frac{t_s M_{2n}}{K}}}. \tag{19}$$

The shorter the starting-up time t_s, the larger the hysteresis gyro motor current, all other conditions being equal.

The power P_{10} used by an idealized hysteresis motor can be expressed by the following equation:

$$P_{10} = P_{2n} + \Sigma\Delta P_0 = P_{2n} + k_1 I_{10}^2 + k_2 B_{\delta_0}^2, \tag{20}$$

where $\Sigma\Delta P = \Delta P_c + (\Delta P_{st} + \Delta P_{sur})$ is the total loss in the motor, $\Delta P_c = k_1 I_1^2$ is the loss in copper, $(\Delta P_{st} + \Delta P_{sur}) = k_2 B_\delta^2$ is the loss in steel (which takes into account the surface losses in the rotor), and k_1 and k_2 are proportionality factors.

For an actual hysteresis motor under the rated conditions (neglecting small changes in the motor current that are due to load variations), the power used is equal to

$$P_1 = P_{2n} + \Sigma\Delta P = P_{2n} + k_1 I_1^2 + k_2 B_\delta^2 \tag{20a}$$

or, by taking into account (18),

$$P_1 = P_{2n} + k_1 k_m I_{10}^2 + k_2 k_m B_{\delta_0}^2 = P_{2n} + k_m \Sigma\Delta P_0 = P_{2n}\left[1 + \frac{1-\eta_0}{\eta_0} k_m\right]. \tag{21}$$

Here, $\eta_0 = \frac{P_{2n}}{P_{10}} = 1 - \frac{\Sigma\Delta P_0}{P_{10}}$ is the relative efficiency of the idealized gyro motor at the calculated point. (According to the footnote (*) on page 1096, the efficiency η_0 and $\cos\varphi_0$ for an idealized gyro motor are equal to the efficiency η_w and $\cos\varphi_w$ of an actual gyro motor at the starting point.)

By using (11), we obtain:

$$P_1 = P_{2n}\left[1 + \frac{1-\eta_0}{\eta_0}\left(1 + \frac{0.725 c_m}{\frac{t_s M_{2n}}{K}}\right)\right] \tag{22}$$

or

$$\frac{P_1}{P_{10}} = 1 + 0.725(1-\eta_0)\frac{c_m K}{t_s M_{2n}}. \tag{23}$$

Thus, with an increase in the starting-up time t_s, the power P_1 used by a hysteresis gyro motor under rated (operating) conditions increases.

Correspondingly, by using (23), (18), and (19), we can obtain the expressions for the duty values of the efficiency η and $\cos\varphi$ for an actual gyro motor:

* Hereafter, the index "0" will pertain to the idealized gyro motor, where $k_m = 1$.

$$\frac{\eta}{\eta_0} = \frac{P_{10}}{P_1} \frac{1}{1 + \frac{0.725\,(1 - \eta_0)\, c_{\text{п}} K}{t_s M_{2\text{п}}}} \tag{24}$$

and

$$\frac{\cos\varphi}{\cos\varphi_0} = \frac{P_1}{P_{10}} \frac{I_{10}}{I_1} \frac{U_{10}}{U_1} = \frac{P_1}{P_{10}} \left(\frac{I_{10}}{I_1}\right)^2 = \frac{1 + 0.725\,(1 - \eta_0)\, \frac{c_{\text{п}} K}{M_{2\text{п}} t_p}}{1 + 0.725\, \frac{c_{\text{п}} K}{t_s M_{2\text{п}}}}\,. \tag{25}$$

It can readily be seen that a decrease in the starting-up time t_s directly leads to worse energy indices (η and $\cos\varphi$) of hysteresis gyro motors under rated (operating) conditions.

Finally, since all of the electric energy used by the gyro motor is transformed into heat energy, which causes the heating of the gyro motor, it is readily seen that the overheating temperature τ° of the gyro motor increases with an increase in the starting-up time t_s:

$$\frac{\tau^\circ}{\tau^\circ_0} = \frac{P_1}{P_{1s}} = 1 + 0.725\,(1 - \eta_0)\, \frac{c_{\text{п}} K}{t_s M_{2\text{п}}}\,. \tag{26}$$

Thus, the presented qualitative analysis of the effect of the starting-up time t_s on the characteristics of hysteresis gyro motors makes it possible to draw definite conclusions, which indicate that, with a decrease in the starting-up time t_s, the duty (operating) values of the current I_1, the power consumption P_1, and the overheating temperature τ° increase, while the efficiency η and $\cos\varphi$ decrease, all other conditions being equal.

LITERATURE CITED

1. V. A. Pavlov, Gyroscopic Aircraft Instruments [in Russian] (Oborongiz, 1954).

All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. *Some or all of this periodical literature may well be available in English translation.* A complete list of the cover-to-cover English translations appears at the back of this issue.

ANALYSIS OF A CIRCUIT FOR CONTROLLING A DC MOTOR BY MEANS OF A BRIDGE REVERSIBLE SEMI-CONDUCTOR AMPLIFIER

G. B. Eliasberg

(Leningrad)

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There is considered the control of a dc motor by means of a reversible amplifier using transistors which operate as switches. Mechanical characteristics and a motor efficiency are determined. Recommendations are given for the choice of a switching pulse frequency.

In recent years there has been intensified interest in the possibilities of controlling dc motors by means of transistors operating as switches. There are several papers [1-3] devoted to this question in which the basic principles of this control method are analyzed, and concrete amplifier circuits are presented.

In [2] there was given a detailed analysis of a nonreversible control circuit for a dc motor. For the purposes of automatic control, however, there is greater interest in circuits which have the possibility of reversing the motor, and also of its operation both as a motor and in a reverse connection as a brake with regeneration. Practically, the most advantageous circuit, which insures meeting the indicated requirements, is the reversible bridge circuit [1, 3]. The present article is devoted to an anslysis of the operations of this circuit. At the same time there is substantial interest in the questions of determining the mechanical characteristics and the efficiency of the motor, and also in the choice of the working frequency. The principle attention in the article is paid expressly to these questions.

1. Basic Operating Principles of the Bridge Reversible Circuit

In Fig. 1 there is presented a bridge circuit for controlling a dc motor by means of transistors. The circuit is controlled with rectangular voltage pulses having a period T and an amplitude V_0 (Fig. 2a). This voltage is fed to the transistor bases in such a way that for the period of time from $t = 0$ to $t = T_1$ transistors Q_1 and Q_2 are conducting and transistors Q_3 and Q_4 are cutoff, while in the period of time $t = T_1$ to $t = T$ transistors Q_3 and Q_4 are conducting and transistors Q_1 and Q_2 are cutoff. By altering the ratio of T_1/T it is possible to regulate the speed of rotation and to reverse the motor. For $T_1/T = 1/2$ the average armature current $I_{av} = 0$, and the motor does not develop a torque. Diodes D1-D4 are provided to ensure uninterrupted commutation (see [3]). The apparatus which shapes the control signal can have a number of forms which are described in the literature (see, for example, [3]). Therefore its operation will not be considered here.

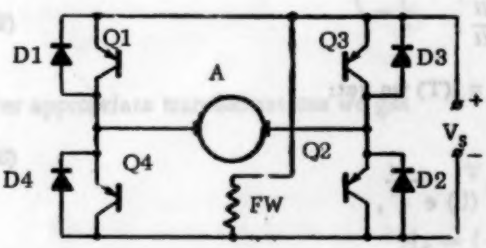

Fig. 1

We will adopt the following assumptions:

a) The current through a transistor which is cutoff, and the switching time are equal to zero;

b) the change in rotational speed over a period T is insignificant and can be neglected;

c) the voltage drop of a transistor when in the saturation region, and also the forward voltage drop of a diode are equal to zero.

For these conditions the parameters of the armature circuit of a motor (its ac resistance and inductance) remain constant during the period T, i.e., they do not depend on the direction and path of the current in the circuit or on the transistor condition.

The equivalent circuit of Fig. 3 can then be substituted for Fig. 1. In this circuit the source of equivalent voltage V_E provides voltage pulses of rectangular shape with an amplitude equal to the supply voltage V_s, a period T, a positive portion of the period of duration T_1, and a negative $T - T_1$ (Fig. 2b). The counter emf is designated by E and the armature current by i.

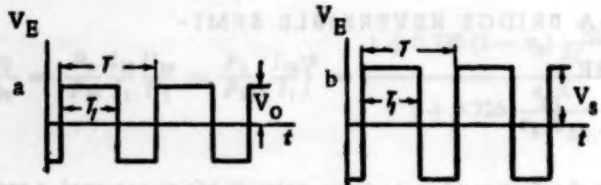

Fig. 2

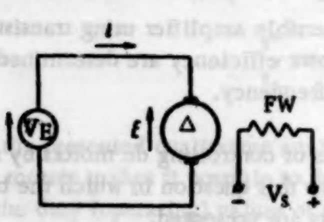

Fig. 3

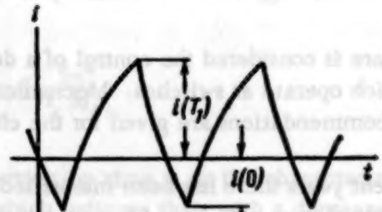

Fig. 4

The arrows indicate the positive directions of the currents and voltages in the circuit of Fig. 3.

For the time interval from $t = 0$ to $t = T_1$ the correct equation is

$$\frac{V_s - E}{r} = i + \tau \frac{di}{dt} \tag{1}$$

where r is the resistance of the armature circuit, τ is the time constant of this circuit.

For the interval from $t = T_1$ to $t = T$ the correct equation is

$$-\frac{V_s + E}{r} = i + \tau \frac{di}{dt}. \tag{2}$$

Solving these equations and taking into consideration that $i(0) = i(T)$ we get:

for the interval from $t = 0$ to $t = T_1$

$$i = \frac{V_s - E}{r}\left(1 - e^{-\frac{t}{\tau}}\right) + i(0)\, e^{-\frac{t}{\tau}}, \tag{3}$$

for the interval from $t = T_1$ to $t = T$

$$i = \frac{V_s + E}{r}\left(1 - e^{-\frac{t-T_1}{\tau}}\right) + i(T_1)\, e^{-\frac{t-T_1}{\tau}}, \tag{4}$$

where

$$i(0) = \frac{V_s\left(2e^{-\frac{T-T_1}{\tau}} - e^{-\frac{T}{\tau}} - 1\right)}{r\left(1 - e^{-\frac{T}{\tau}}\right)} - \frac{E}{r}, \tag{5}$$

$$i(T_1) = \frac{V_s\left(e^{-\frac{T}{\tau}} - 2e^{-\frac{T_1}{\tau}} + 1\right)}{r\left(1 - e^{-\frac{T}{\tau}}\right)} - \frac{E}{r}. \tag{6}$$

It is obvious that i(0) is the minimum value of the armature current during the period T, and $i(T_1)$ is its maximum value. In the time intervals between these values the armature current changes according to an exponential law. A characteristic curve is presented in Fig. 4.

The values of i(0) and $i(T_1)$, each taken separately, can assume either positive or negative values depending on the circuit parameters and the ratio T_1/T.

2. Mechanical Characteristics and Motor Efficiency

Expressions (3) and (4) are cumbersome and inconvenient for further study. Therefore it is expedient to make some simplifications which permit the calculations to be simplified while producing little effect on the final results.

Let us introduce the voltage V_E in the form of a Fourier Series:

$$V_E = V_{AV} + \sum_{k=1}^{\infty}\left(A_k \sin\frac{2\pi kt}{T} + B_k \cos\frac{2\pi kt}{T}\right), \tag{7}$$

where

$$V_{AV} = \frac{1}{T}\left(\int_{-\frac{T}{2}}^{-\frac{T}{2}+T_1} V_s\, dt - \int_{-\frac{T}{2}+T_1}^{\frac{T}{2}} V_s\, dt\right), \tag{8}$$

$$A_k = \frac{2}{T}\left(\int_{-\frac{T}{2}}^{-\frac{T}{2}+T_1} V_s \sin\frac{2\pi kt}{T}\, dt - \int_{-\frac{T}{2}+T_1}^{\frac{T}{2}} V_s \sin\frac{2\pi kt}{T}\, dt\right) \tag{9}$$

$$B_k = \frac{2}{T}\left(\int_{-\frac{T}{2}}^{-\frac{T}{2}+T_1} V_s \cos\frac{2\pi kt}{T}\, dt - \int_{-\frac{T}{2}+T_1}^{\frac{T}{2}} V_s \cos\frac{2\pi kt}{T}\, dt\right). \tag{10}$$

After appropriate transformations we get

$$V_{AV} = V_s\left(2\frac{T_1}{T} - 1\right), \tag{11}$$

$$A_k = (-1)^k \frac{2V_s}{\pi k}\left(1 - \cos\frac{2\pi kT_1}{T}\right), \tag{12}$$

$$B_k = (-1)^k \frac{2V_s}{\pi k} \sin\frac{2\pi kT_1}{T}. \tag{13}$$

The average value of the armature current is, in a similar way

$$I_{AV} = \frac{V_{AV} - E}{r} = \frac{V_s\left(2\frac{T_1}{T} - 1\right) - E}{r}. \tag{14}$$

Expression (14) is analogous to the equation for the armature circuit of a motor being controlled by a dc voltage V if we set $V = V_s(2T_1/T-1)$. Therefore the mechanical characteristics of the motor are identical for both control methods.

When $T_1 = T/2$, $V_{AV} = 0$ and the motor is operating in a regenerative braking condition with increased power losses on account of the pulsations.

From the power point of view it is of interest to determine the effective value of armature current I_{Eff} and the motor efficiency $K_e = I_{AV}/I_{Eff}$.

Designating by I_k the effective value of the kth harmonic of the current i, by V_k the amplitude of the kth harmonic of the voltage V_E, by Z_{Ak} the impedance of the armature circuit for the kth harmonic, we get

$$I_k = \frac{U_k}{\sqrt{2}\, Z_{Ak}}, \tag{15}$$

where

$$V_k = \sqrt{A_k^2 + B_k^2}, \tag{16}$$

$$Z_{Ak} = r\sqrt{1 + \left(\frac{2\pi k\tau}{T}\right)^2}. \tag{17}$$

Taking into account (12) and (13) we get from (16), after some intermediate transformations

$$V_k = \frac{4\,V_s}{\pi k}\sin \pi k q, \tag{18}$$

where

$$q = \frac{T_1}{T}.$$

It is shown below that in order to get an acceptable value for efficiency, it is necessary to select a period for the control volt which is of the same order as the time constant, or even less. Taking this into account expression (17) can be simplified:

$$Z_{Ak} \approx \frac{2\pi k\tau r}{T}. \tag{19}$$

Let us find the value of the ratio between the current of the kth harmonic I_k and the current of the first harmonic I_1. Taking into account (15), (18), and (19) we get

$$\frac{I_k}{I_1} = \frac{V_k Z_{A1}}{V_1 Z_{Ak}} = \frac{1}{k^2}\,\frac{\sin \pi k q}{\sin \pi q}. \tag{20}$$

Study of expression (20) shows that for any values $0 < q < 1$ the following relationship is true

$$\frac{I_k}{I_1} < \frac{1}{k}. \tag{21}$$

The ratio of heat losses, determined by their corresponding current harmonics, satisfy the inequality

$$\frac{P_k}{P_1} = \frac{I_k^2}{I_1^2} < \frac{1}{k^2}. \tag{22}$$

Expression (22) shows that when determining motor efficiency it is possible to neglect all current harmonics except the first without much error.

We obtain the value of current I_1 from (15), (18), and (19):

$$I_1 = \frac{\sqrt{2}\, V_s T}{\pi^2 \tau r} \sin \pi q. \tag{23}$$

Knowing the value of the average armature current I_{AV} and having found from (23) the value of the first harmonic current, the efficiency can be found from the formula

$$K_e = \frac{I_{AV}}{\sqrt{I_{AV}^2 + I_1^2}}\,, \tag{24}$$

3. Selection of the Control Voltage Frequency

Expression (23) shows that for any given value of q the current I_1 does not depend on either the load or the rotational speed. For q = 0.5, i.e., in the dynamic braking condition, the value of I_1 is a maximum. Moreover, the value of I_1 is inversely proportional to the frequency of the control voltage $f = 1/T$. Therefore, for the best motor efficiency it is desirable to increase the frequency. However, as is well known, this brings about an increase of the switching losses in the transistors and additional heating in them. Therefore the frequency should not be increased above the value which is required to give an acceptable motor efficiency.

The determination of the required value of f must be directed towards the operating condition of the motor for which the effective value of the armature current is a maximum. For example, in servo system motors such an operating condition is represented when the signal at the input of the system amplifier is somewhat less than the signal which starts it in motion. In this instance the armature current is close to the starting current but the speed is equal to zero. Such a condition, generally speaking, can exist for a long period. We will assume that the starting current I_{ST} is equal to the nominal armature current I_{NOM}. Generally speaking, in servo systems $I_{ST} < I_{NOM}$, as a rule. However, in practice cases are encountered when $I_{ST} \approx I_{NOM}$. Obviously such a case corresponds to the most difficult operating condition of a motor, and therefore it is expedient to consider it expressly as an example.

Let us denote by K_{ST} a multiplier for the starting current

$$K_{ST} = \frac{V_s}{r I_{NOM}}. \tag{25}$$

From (14) and (25) we have for the above-mentioned condition

$$I_{AV} \approx I_{NOM} = \frac{V_s}{r K_{ST}} = \frac{V_s \left(2\frac{T_1}{T} - 1\right)}{r}. \tag{26}$$

Whence we find

$$q = \frac{T_1}{T} = \frac{1 + \frac{1}{K_{ST}}}{2}. \tag{27}$$

For a dc motor the value of K_{ST} is generally not less than 5-10. Therefore the value of q as determined from expression (27) is close to 0.5.

Hence for the operating condition being considered the first harmonic of the armature current I_1 must have a value close to the maximum at the given frequency. Since $I_{AV} \approx I_{NOM}$, it is obvious that this condition is the most difficult of all those possible.

Supposing, for example, that the allowable value of efficiency K_e is equal to 0.9, we find from (24)

$$I_1 = \frac{I_{AV}}{K_e} \sqrt{1 - K_n^2} \approx 0.48\ I_{AV}. \tag{28}$$

Substituting (27) and (28) in (23) and taking into account (26) we get

$$0.48\frac{V_s}{r\,K_{ST}} = \frac{\sqrt{2}V_s\,T}{\pi^2\tau r}\,\sin\pi\,\frac{1+\frac{1}{K_{ST}}}{2}, \tag{29}$$

whence

$$\frac{T}{\tau} = \frac{0.48\pi^2}{K_{ST}\,\sqrt{2}\sin\dfrac{\pi\left(1+\frac{1}{K_{ST}}\right)}{2}}. \tag{30}$$

Expression (30) determines the value of the period T, or the frequency $f = 1/T$, which is required in order to obtain an acceptable motor efficiency.

To increase the frequency above this value is not expedient because it brings about an increase in the transistor losses. It should be noted that the value of T which is found from expression (30) is close to the value of the time constant. For example, when $K_{st} = 5$, we get $T = 0.7\tau$ from (30).

It must be emphasized that expression (30) determines the frequency only for the particular case, given as an example, when $I_{ST} \approx I_{NOM}$, $K_{ST} = 5$, and $K_e = 0.9$. In the general case, knowing the armature parameters (τ and r), and also the current I_{AV} for the worst condition, and assuming a permissible value for K_e, these values must be substituted in expressions (23) and (24) and, by a common solution obtained of the equations, the required value of f is then found.

SUMMARY

1. The mechanical characteristics of a motor which is controlled by means of a bridge amplifier using transistors does not differ in principle from the mechanical characteristics of a motor which is controlled by means of a change in the voltage supplied to the armature.

2. When choosing the frequency of the control pulses the value of motor efficiency found from expressions (23) and (24) must come within acceptable limits. The frequency must not be increased above the value determined by these expressions.

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All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. ***Some or all of this periodical literature may well be available in English translation.*** **A complete list of the cover-to-cover English translations appears at the back of this issue.**

ON THE THEORETICAL POSSIBILITY OF CARRYING OUT A PRACTICALLY INERTIALESS TEMPERATURE MEASUREMENT OF GASES AND LIQUIDS BY MEANS OF VERY SIMPLE PNEUMATIC AND HYDRAULIC SENSORS

L. A. Zalmanson
(Moscow)

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It is shown that there is a possibility for carrying out practically inertialess temperature measurements of gases and liquids in the following way: the temperaturesare taken while the fluids are flowing through a test chamber which is provided at the inlet and outlet with flow impedances having characteristics (flow versus pressure differential) that vary with temperature in a different way. An analysis of the flow characteristics through such a chamber is carried out. Methods for accounting for and compensation of the errors are given which are caused by heat exchange of the fluids with the walls of the test chamber.

Ordinary pneumatic and hydraulic temperature sensing devices which have widespread technical applications, specifically in automatic control systems, are suitable only for measurement of stationary or slowly changing temperatures. The explanation for this is that the basic principle of their operation is functionally connected with heat transfer from the gas or liquid to the container walls and through the container walls.

As a characteristic example, a schematic sketch of one of the pneumatic temperature measuring devices of this type [1, 2] is reproduced in Fig. 1a. The gas whose temperature T_0 is to be measured is flown through a measuring tube in which are mounted the flow impedances 1, 2, and 3 (orifices, baffles, valves, filters, etc.) and the exchanger 4. The temperature of the gas T_0 being tested is judged by the difference in pressure δp_1 before and after flow impedance 1. This proves to be possible, if the mass flow of gas in the measuring tube is kept constant. This is made sure of by maintaining the pressure differential δp_2 across impedance 2 constant by adjustment of the open passage area of impedance 3, and also by keeping the temperature T_1 of the gas reaching 2 constant; the latter is accomplished by cooling of the gas from its temperature T_0 to the temperature T_1 by means of the heat exchanger 4. Heat exchangers or heat accumulators of one kind or another are unavoidable accessories of other pneumatic and hydraulic temperature sensing devices also [3 to 6 and others].

Of special interest is the principle of temperature measurement described in [7], even though the form of measuring apparatus outlined in that paper is suitable only for measurement of stationary temperatures. A diagramatic sketch of this apparatus is shown in Fig. 1b. Change of pressure in chamber 1 as a function of temperature of the flowing-through gas is here obtained by the use at its ends of flow impedances of two types having radically different temperature characteristics: one of these–2–is a flow impedance producing laminar flow, consisting of a plate of porous material; the other–3–is a flow impedance creating turbulent flow. Ahead of impedance 2, a filter 4 is installed. To maintain the walls of chamber 1 at a temperature equal to that of the gas flowing through the chamber, the gas is passed at the same temperature through jacket 5 surrounding chamber 1, where it is continuously flowing over the outside of the chambers walls. Inasmuch as the characteristics of this device too are influenced by the temperature of the walls, the process taking place is just as much subject to inertia as in other known pneumatic temperature sensing devices.

It is shown below that the system with two flow impedances of different type can serve as a basis for development of methods for practically inertialess measurements of momentary values of the temperature of gases and liquids in nonstationary processes, when their composition is known and fixed.

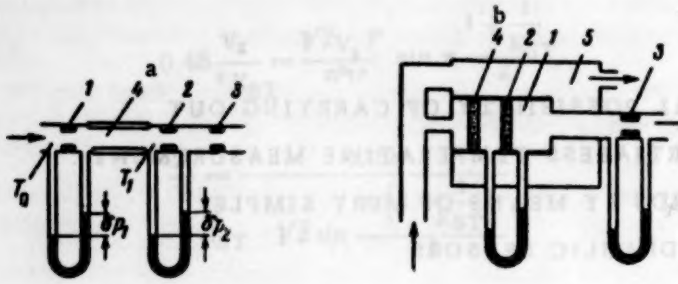

Fig. 1

1. Analysis of the Basic Characteristics of Pneumatic and Hydraulic Chambers Terminated by Flow Impedances of Different Type when Used as Temperature Sensors

1. Initial premises. Preliminary considerations regarding the use of chambers of the described type for practically inertialess measurement of nonstationary temperatures. Let us examine a chamber which is provided at its inlet and its outlet with flow impedances of different type. Let, for instance, the impedance at the inlet be of a type causing turbulence, and the one at the outlet of a type providing laminar flow (Fig. 2). The mass flows of a

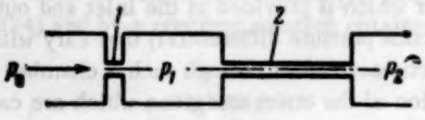

Fig. 2

gas or a liquid through these two impedances have different temperature dependences. To illustrate the prevailing conditions, in Fig. 3a characteristic curves are given which show the change with temperature of the mass of air passed per second through a turbulence type impedance and through a laminar type impedance. The ordinates of these curves give the ratios of the mass throughput at the corresponding temperatures on the abscissa to the mass-throughput at 15°C under the same conditions. In Fig. 3b are shown experimental data obtained when water was passed through the two different flow impedances; however, here, the throughputs per second are plotted with reference to the throughput at 8°C, instead of 15°C as in the case of air. In both Fig. 3a and Fig. 3b, the curves marked 1 were obtained with turbulent flow, and those marked 2 with laminar flow.

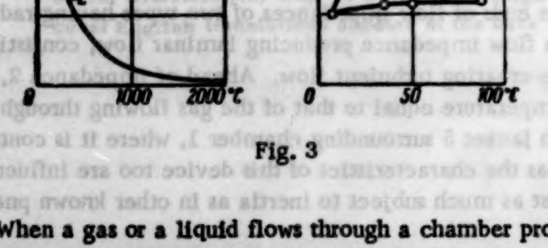

Fig. 3

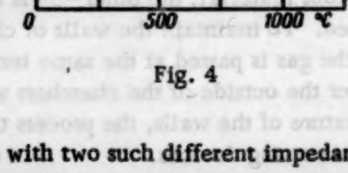

Fig. 4

When a gas or a liquid flows through a chamber provided at its ends with two such different impedances (Fig. 2), then the pressure p_1 in the chamber—with the pressures p_0 and p_2 remaining constant—varies as a function of the temperature flowing through the chamber medium. And here heat exchange through the walls is no more a necessary element of the temperature measurement process. On the basis of the above reasoning, in [7] the conclusion is

reached that the use of a chamber with different kinds of impedances at the ends is advantageous for increasing precision in determinations of stationary temperatures under conditions of small gas throughputs.

If it is possible under actual conditions to achieve the chamber characteristics with respect to absence of heat exchange with the walls, indicated in this section, or to eliminate the effect of heat exchange altogether by other means (see section II), then it is also possible to measure temperatures of gases and liquids in nonstationary, fast going processes by means of a chamber terminated by two flow impedances of different type. Indeed, when the effect of heat transfer upon the function $p_1 = \varphi(T)$ is eliminated, then the time constant of the temperature sensor is determined only by the velocity of the liquids flow through the orifices of the flow impedance and through the chamber. In the case of gases, the time constant depends also upon the speed with which the pressure in the chamber is built up. However, when the dimensions of the chamber are sufficiently small, the time constant – with all these processes present – still is of the order of a hundredth's or a thousandth's part of second [8].

2. Calculation and analysis of the relationship $p_1 = \varphi(T)$ for gases. Suppose that in the chamber shown in Fig. 2 the flow impedances 1 and 2 are of such construction and the pressures p_0 and p_2 are so chosen, that the first of these impedances is of the turbulence producing kind, the second – of the laminar flow type.

We shall use formulas (I, 10), (I, 11), (I, 26) and (I, 83) of paper [8]. Using these formulas, we consider the processes in the flow impedances and in the chamber to be quasistationary. We must remark that formula (I, 26) is valid for isothermic conditions, while, as the problem is set up here, the flow should be considered of an adiabatic nature. However, there will be no serious error incurred by using formula (I, 26), because, when the pressure drops involved are small, the throughputs obtained at equal pressure differentials are practically the same in both kinds of flow.

Using the indicated formulas, we write down the equation expressing equality of the quantity of material passing through the two impedances per unit time – twice: once for the normal air temperature (288°K = 15°C), and the second time for the temperature of the gas under test T. Designating the pressures in the chamber that correspond to these two temperatures respectively by p_{1*} and p_1, we obtain from the above two equations the following expression:

$$\varphi\left(\frac{p_1}{p_0}\right) = \varphi\left(\frac{p_{1*}}{p_0}\right)\psi(T), \tag{1}$$

where when the flow through impedance 1 is below the critical value we have:

$$\varphi\left(\frac{p_1}{p_0}\right) = \frac{(p_1/p_0)^2 - (p_2/p_0)^2}{\sqrt{(p_1/p_0)^{2/k} - (p_1/p_0)^{(k+1)/k}}}, \tag{2}$$

and when the flow in 1 is above the critical, we have:

$$\varphi\left(\frac{p_1}{p_0}\right) = \frac{(p_1/p_0)^2 - (p_2/p_0)^2}{\sqrt{\frac{k-1}{k+1}\left(\frac{2}{k+1}\right)^{2/(k-1)}}}. \tag{2'}$$

From the above we derive:

$$\psi(T) = (121 + 1{,}62T)\sqrt{T}\cdot 10^{-4}. \tag{3}$$

The curve representing the function $\psi(T)$, plotted according to this formula, is presented in Fig. 4. However, while the temperature T in the above formulas is expressed on the absolute scale, in Fig. 4 the temperature is expressed in centigrades.

Curves giving the values of $\varphi(p_1/p_0)$ for the entire range of feasible flow conditions through the impedance 1 and for a number of p_2/p_0 ratios, as calculated by formulas (2) and (2') with k = 1.4, are presented in Fig. 5a. Solution of equation (1) above with the help of these curves is carried out as follows: First we find, using Fig. 5a, the value of $\varphi(p_{1*}/p_0)$ which corresponds to the pressure p_{1*} (known from the setup of the problem), and then we multiply it by the values of $\psi(T)$ corresponding to a number of chosen values of T taken from Fig. 4. From the obtained in this manner values of the quantity $\varphi(p_1/p_0)$ [see equation (1)] we obtain the corresponding values of p_1/p_0 by using a suitable curve of Fig. 5a, and thus, since we know p_0, we have found the values of p_1 corresponding to the chosen values of T.

When interpolating for values of p_2/p_0 between those for which curves are given in Fig. 5a, the following consideration should be kept in mind which will make plotting of intermediate curves easier: each of the $\varphi(p_1/p_0)$ curves intersects the abscissa axis at a point whose distance from the origin is equal to the p_2/p_0 ratio of this very curve.

Let us examine a numerical example. Let us have for the chamber of Fig. 2 the following parameters: $p_0 = 1.5$ atm (absolute), $p_2 = 1$ atm (absolute), and $p_{1*} = 1.1$ atm (absolute). Let us determine the function $p_1 = \varphi(T)$ which will be valid when the flow impedances at the ends of the measuring chamber are of the above indicated character. Carrying out the computation by the described graphic method we obtain the relationship $p_1 = \varphi(T)$ shown as curve 1 in Fig. 5b.

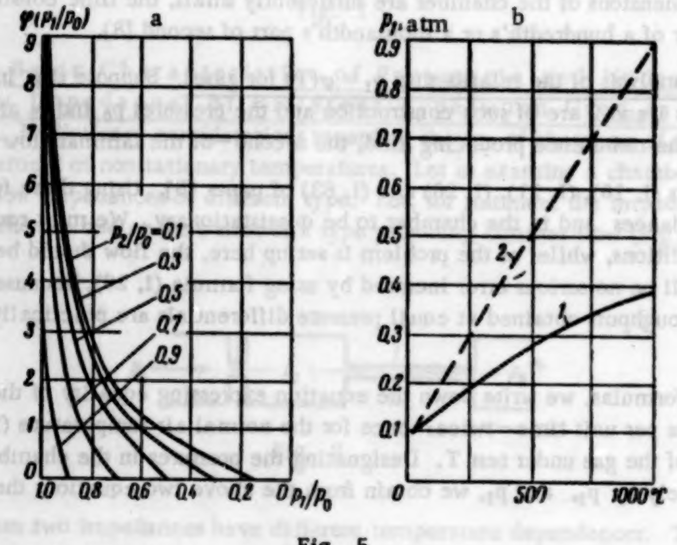

Fig. 5

The steepness of the curve $p_1 = \varphi(T)$ depends substantially upon the value of p_0: when, leaving the values of p_2 and p_1 the same as in the preceding example, the value of p_0 is increased to $p_0 = 5$ atm, we obtain for $p_1 = \varphi(T)$ the much steeper curve 3 of Fig. 2. It should be noted that, to achieve this with all other characteristics remaining constant, it is necessary to decrease the cross section area of flow impedance 1.

All given above data pertain to the arrangement of the flow impedances of diverse type 1 and 2 shown in Fig. 2. However, the reverse arrangement of these flow impedance is also feasible, that is, having the laminar impedance in the inlet, and the turbulent in the outlet. The formulas for computation and the auxiliary curves for this case can be obtained in a manner similar to the one used above in connection with the arrangement shown in Fig. 2.

3. Calculation of the characteristics of $p_1 = \varphi(T)$ for a liquid. Let us examine as before the chamber shown in Fig. 2. Let the flow through impedance 1 be a turbulent one, and let us suppose that for this particular impedance the local resistances at the inlet and the losses at the outlet are of prevalent importance; let the flow through impedance 2 be a laminar one, and in it let the friction losses in the channel be of major importance.

The throughput of liquid through flow impedances 1 and 2 are computed respectively by the formulas:

$$G = \varepsilon f \sqrt{2g\gamma}\,\sqrt{p_0 - p_1}$$

and

$$G = \frac{\pi d^4 \gamma}{128\, l}(p_1 - p_2)\,\frac{1}{\mu}.$$

Equating the two throughputs expressed by these formulas, we obtain:

$$\frac{\sqrt{p_0 - p_1}}{p_1 - p_2} = \frac{\pi d^4 \sqrt{\gamma}}{128\, l\, \varepsilon f \sqrt{2g}}\,\frac{1}{\mu}. \tag{4}$$

In distinction from gases, in dropforming liquids viscosity does not increase with temperature, but decreases, and in addition their specific gravity changes less with temperature than that of gases. As an example, in Fig. 6a are shown curves expressing the change with temperature of the viscosity μ of water (curve 1) and of a motor oil (curve 2); (viscosity is here measured in practical metric units of kg-wt-m^{-2}sec, translator). On the same figure another curve (3) is given, which shows the change of specific gravity with temperature for water. This is taken from [9]. According to the data of Fig. 6a, the quantity $\sqrt{\gamma}$ (γ = sp. gr.) for water decreases by less than 2%, while μ is decreased by a factor of 6.5, when the temperature increases from 0 to 100°C.

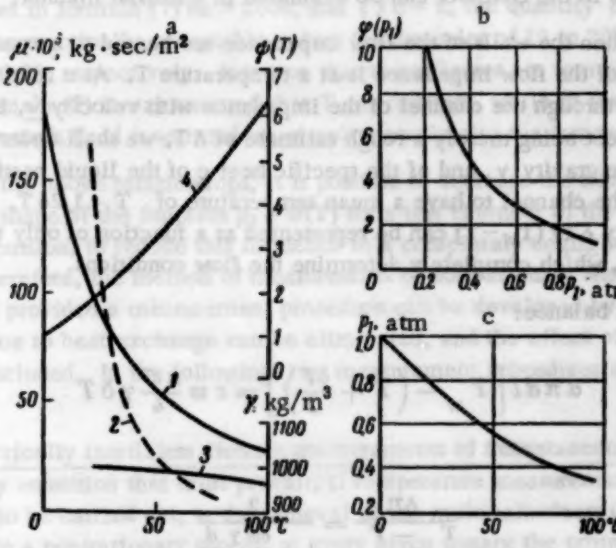

Fig. 6

Considering therefore $\sqrt{\gamma}$ in expression (4) as one of the constants, and taking into account the thus found dependence of μ upon temperature we obtain from (4) the following equation:

$$\frac{\sqrt{p_0 - p_1}/(p_1 - p_2)}{\sqrt{p_0 - p_{1*}}/(p_{1*} - p_2)} = \psi(T). \tag{5}$$

where the asterisk added to the subscript of p_1 now indicates the value of p_0 at 0°C (and not at 15°C as in the preceding part).

The curve representing the function $\psi(T)$ for water is shown as curve 4 in Fig. 6a.

Equation (5) contains only ratios of pressure differences; therefore, this equation is valid, regardless whether all three pressures p_0, p_1, and p_2 are measured in absolute units, or as the excess values above atmospheric pressure, or in any other units.

Let us examine the following example: through a system of flow impedances arranged as shown in Fig. 2, water is flowing. The following pressure conditions are in effect: p_0 = 1.2 atm, p_{1*} = 1 atm, and p_2 = 0 atm. With these values of p_0, p_{1*}, and p_2, formula (5) is reduced to the form of: $2{,}24\sqrt{1{,}2-p_1}\,/\,p_1 = \psi(T)$. The curve expressing the function $\varphi(p_1) = 2{,}24\sqrt{1{,}2-p_1}\,/\,p_1$ is shown in Fig. 6b. Chosing a number of different values of T, we take the values of $\psi(T)$ corresponding to these values of T from curve 4 of Fig. 6a. Then equating these values with the values of $\varphi(p_1)$ expressed by the curve of Fig. 6b, we find the values of p_1 corresponding to the chosen values of T, and thus obtain the curve $p_1 = \varphi(T)$ shown in Fig. 6c. We see that with increase of temperature the value of p_1 decreases, contrary to what is taking place in gas flow with the same arrangement of flow impedances. This is explained by the pointed out above opposite effects of temperature upon viscosity in gases and in liquids. When the arrangement of the flow impedances is reversed (the laminar impedance in the inlet, and the turbulent in the outlet), then for a liquid the value of p_1 must increase with temperature.

II. Consideration of Errors Caused by Heat Exchange with the Walls, and Their Compensation

1. Conditions for decrease of errors connected with heat exchange. Let us estimate the temperature change of the working substance caused by heat transfer to it from the walls. We shall examine flow of liquids; however, the results of our examination can be applied also for an approximate analysis of conditions in the case of gas flow.

A nearly perfect turbulent flow impedance can be made in the shape of an orifice in a thin plate; and the flow through such an orifice is practically adiabatic. More substantial processes of heat exchange take place in the laminar type flow impedance and in the chamber between the two flow impedances. We shall limit our examination to the heat transfer from the walls to the liquid in a laminar type impedance having a channel of diameter d and length l. The conditions of heat exchange in the chamber can be evaluated in a similar manner.

Let us analyze the case when the walls of the flow impedance are heated to a constant temperature T_w, and the liquid entering the channel of the flow impedance is at a temperature T. As a result of heat transfer from the walls, the liquid, while flowing through the channel of the impedance with velocity w, is heated, and its temperature increased by δT. Our object being merely a rough estimate of δT, we shall undertake to find only the constant mean values of the specific gravity γ, and of the specific heat c of the liquid passing through the channel; we shall consider the liquid in the channel to have a mean temperature of $T + 1/2\delta T$. We will show that under the above conditions the quantity $\delta T/(T_w - T)$ can be represented as a function of only two parameters: the ratio l/d and the Reynolds number Re, which completely determine the flow conditions.

From the equation of heat balance:

$$\alpha \pi d l \left[T_w - \left(T + \frac{\delta T}{2} \right) \right] = c\, w \frac{\pi d^2}{4} \gamma \delta T$$

we obtain the following:

$$\frac{\delta T}{T_w - T} = \frac{2}{1 + \frac{c w \gamma}{2\alpha} \frac{d}{l}}. \tag{6}$$

In the above formula, α is the coefficient of heat transfer, which in the case of laminar flow can be represented in the form of the following function of Re, c, γ, and w:

$$\alpha = \frac{8}{Re} c \gamma w.$$

This has been derived from the general relationship between α and the coefficient of friction losses ξ_{fr} when a liquid flows through a channel with heat exchange, which is: $\alpha = \frac{\xi_{fr}}{8} c\gamma w$ (see [10]).

Substituting the above expression for α in terms of the Reynolds number into (6), we obtain the following simple formula for determination of $\delta T/(T_w - T)$ from the given values of l/d and Re:

$$\frac{\delta T}{T_w - T} = \frac{2}{1 + \frac{1}{16} \frac{Re}{l/d}}. \tag{7}$$

It follows from (7) that to reduce δT to a minimum, the values of Re used in these operations must be as close as possible to the upper limit Re_{lim} of the Reynolds number beyond which laminar flow is not possible. On the basis of (7) it would also appear that l/d must be chosen as small as possible. This seems to be feasible, since to secure laminar flow at small ratios of l/d, it is only necessary to have a sufficiently small d*. The desirable characteristics

* It follows from the conclusions in paper [8] that the limiting value of d at which laminar flow in an impedance changes to turbulent flow is $d = 270 \frac{\mu}{\sqrt{\rho \delta p}} \sqrt{\frac{l}{d}}$, where μ is the dynamic coefficient of viscosity, ρ – the density, and δp – the pressure differential across the impedance; all quantities are measured in units of the kg-m-sec systems.

can be obtained by using for the laminar type impedance a honeycomb construction with a large number of parallel channels. A flow impedance made of a porous material comes very close to a honeycomb flow impedance in its characteristics.

However, when the value of l / d becomes very small, in the present application another consideration enters the picture: when l/d is very small, the process of impeding the flow is carried out not so much by friction losses in the channel, as by localized losses of the total pressure at the entrance to the impedance device and by losses at the exit from it; and when this condition is reached, then the dependence of material throughput upon temperature which was the basis of our deductions in section I above is no longer valid. Therefore, actually the relative temperature change of the fluid under test while passing through the laminar flow impedance cannot be made very small. For instance if we set in formula (7) Re = 2000, and $l / d = 2$, the quantity $\delta T/ (T_W - T)$, will amount only to about 0.03; but, if l/d has practically acceptable values (of the order of 10 to 20), the value of this quantity goes up already to from 0.15 to 0.28, respectively. It is true that these figures are somewhatexaggerated due to the fact that in the deduction of formula (7) it was assumed that T_W is constant, while in reality under nonstationary conditions, when the passing through fluid is warmed up, simultaneously also the temperature of the walls is decreased.

Taking into account the above relationships, it is possible to decrease the adverse influence of heat transfer upon obtaining a desirable shape of the function $p_1 = \varphi(T)$ for a test chamber of the described type. However, as follows from the above discussion, to reduce this influence to a completely negligible extent merely by proper dimensioning is not possible. Therefore, the method of measurement of nonstationary temperatures examined in this paper will be advantageous only, provided a measurement procedure can be developed by which the multiple-valuedness of the function $p_1 = \varphi(T)$ due to heat exchange can be eliminated, and the effect of heat transfer as a factor slowing down the process can be excluded. In the following, two measurement procedures are presented which fulfill these conditions.

2. <u>A method of practically inertialess discrete measurements of instantaneous temperature values in gases and liquids.</u> The first necessary condition that must prevail, if temperature measurements using the principle exposed in section I of this paper are to be carried out, is the removal of the multivaluedness of the function $p_1 = \varphi(T)$, which is caused by the fact that in a nonstationary process at every given instant the temperatures of the conduit walls may be different, even though the fluid passing through the inside is at a certain given temperature. In order for the function $p_1 = \varphi(T)$ to be strictly singlevalued in the presence of heat exchange between the fluid and the walls, it is sufficient that the walls be at a certain fixed temperature at the start of every measurement, and that the entire measurement be completed in a certain predetermined time. The temperature of the walls can be maintained practically constant, if the measurements are carried out not continuously, but in such a way that the flow through the chamber of the fluid under test is switched on only for short intervals of time, and sufficiently long intermissions are left between these intervals for the wall temperature to be restored to the fixed initial value. This restoration may be carried out by an external heat source or sink. For instance, in the setup shown in Fig. 1b, the external jacket surrounding the chamber may be separated from the test fluid flow, and a cooling liquid passed through it. However, it is more advantageous to use internal flushing of the test chamber by a cooling substance, and if the flushing is done in reverse to the direction of the test flow, then in addition to the cooling, the function of cleaning the chamber and flow impedances of contaminating particles is carried out. Under these conditions, the characteristic function $p_1 = \varphi(T)$ can be computed with precision taking into account the thermal properties of all used materials. This function can also be made more precise by laboratory calibration of the sensor with the aid of optical temperature measuring devices or high sensitivity thermocouples, which are unsuitable for continuous operation under field conditions. Such a calibrated pneumatic or hydraulic sensor can be used for measurement of instantaneous temperature values under field conditions. By using several such sensor chambers, switched on one after another, the intermissions between successive measurements can be made as short as desired.

The offered here method of temperature measurement is of interest in connection with the general development of discrete, step by step techniques in control and management of operations.

3. <u>Compensation of errors connected with heat transfer in continuous measurements.</u> As was already pointed out, in the case of continuous measurements, the transient wall temperatures can be different for the same temperatures of the fluids being tested. However, in this case also, the measurement errors connected with heat transfer can be excluded. For this, it is only necessary to make a proper selection of the material of the sensor chamber or of one of the flow impedances: with this done, when the walls are subjected to a warming up by a heat transfer which would change the value of p_1, a change of crossection of the passage area of one or both of the flow im-

pedances by heat expansion or contraction will occur which will compensate the effect of the heat transfer. For example, since the cross section of the passage through flow impedance 2 (Fig. 2) increases with increase of the wall temperature, the increase of p_1 which would take place because of heat transfer from the walls to the test fluid if the passage area would be unaltered, is compensated for by a decrease of p_1 because of the increase of passage area of the given impedance due to heat expansion. In practice it may prove more convenient to carry out this compensation in the turbulent flow impedance.

Both opposing processes, the temperature change of the walls affecting the heat exchange with the fluid under test and the change of passage cross section of the flow impedance, proceed slowly. However, in view of the fact that these two factors are interconnected and act simultaneously but in opposite directions, for the fast process represented by the function $p_1 = \varphi(T)$ the compensation can be complete if the parameters are chosen correctly.

In conclusion, we will point out that in this paper only the principle of this method of temperature measurement of gases and liquids was described, and the feasible design modifications of suitable pneumatic or hydraulic transducers were not analyzed. For instance, depending on the range of temperatures to be measured, there will have to be a choice made of suitable materials for the components. To obtain sufficiently high material-throughput, it may be advantageous to install a honeycomb cell for the laminar flow impedance, or the part of the laminar impedance may be carried out by the walls of the chamber if they are made of a porous material. It may prove advantageous to install a special compensating element (for instance, a needle, inside of the flow impedance channel) for the above discussed compensation of errors. Special consideration must be given to the problems of filtrations etc. All these problems must be solved in connection with the actual technical conditions.

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THE STABILITY OF A CLASS OF SYSTEMS WITH VARIABLE PARAMETERS WHICH VARY PERIODICALLY AND STEPWISE

I. V. Pyshkin

(Moscow)

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A characteristic equation is derived from which the stability of a class of variable parameter systems is determined, the parameters varying periodically and stepwise. The equation can also be used to investigate the stability of periodic oscillations in nonlinear systems with simple nonlinearities.

The systems with variable parameters which vary periodically and stepwise were investigated in a number of papers [1, 2] and the conclusions were drawn that the investigation of such systems in their general form leads to extremely involved relations. In [3, 4] special cases were considered of systems with variable parameters, in particular, systems with a switchkey which can be regarded as ones whose feedback coefficient varies periodically and stepwise.

In the present paper a method is given which is based on the results obtained in [3] when investigating the stability of a class of systems with variable parameters, the latter varying periodically and stepwise. The characteristic equation which determines the stability of this system with variable parameters, that being the most encountered case, is obtained in a compact form.

The obtained results can also be used in the investigation of the stability of auto-oscillations in systems which contain simple broken-line nonlinearities, in particular, those with saturation.

The system of simultaneous equations describing the behavior of a linear control system in the absence of a disturbing force, can be given in the form

$$\frac{d\varphi_k}{dt} = \sum_{\alpha=1}^{m} b_{k\alpha}(t)\,\varphi_\alpha \qquad (k = 1, 2, \ldots, m). \tag{1}$$

In the case of constant coefficients, in order that the corresponding system be stable, it is necessary that the roots of the characteristic equations

$$\begin{vmatrix} \mu - b_{11} & -b_{12} & \ldots & -b_{1m} \\ -b_{21} & \mu - b_{22} & \ldots & -b_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ -b_{m1} & -b_{m2} & \ldots & \mu - b_{mm} \end{vmatrix} = 0 \tag{2}$$

have negative real parts.

We shall consider a class of systems with variable parameters, the latter varying both periodically and stepwise; in these systems the increment $\Delta b_{k\alpha}$ of an arbitrary coefficient $b_{k\alpha}$ can be represented in the form of a product of two sequences of constant quantities h_k and γ_α ($k, \alpha = 1, 2, \ldots, m$) such that

$$\Delta b_{k\alpha} = h_k \gamma_\alpha, \tag{3}$$

i.e.,

$$b_{k\alpha}(t) = \begin{cases} b_{k\alpha} & \text{for} \quad nT < t < nT + T_1, \\ b_{k\alpha} + h_k\gamma_\alpha & \text{for} \quad nT + T_1 < t < nT + T_1 + T_2 = (n+1)\,T. \end{cases} \tag{4}$$

The system of differential equations (1) with variable parameters can be written in this case as

$$\frac{d\varphi_k}{dt} - \sum_{\alpha=1}^{m} b_{k\alpha}\varphi_\alpha + f(t)\,h_k \sum_{\alpha=1}^{m} \gamma_\alpha\varphi_\alpha \qquad (k = 1, 2, \ldots, m), \tag{5}$$

where

$$f(t) = \begin{cases} 0 & \text{for} \quad nT < t < nT + T_1, \\ 1 & \text{for} \quad nT + T_1 < t < nT + T. \end{cases} \tag{6}$$

It was shown in [3] that the characteristic equation which determines the stability of the system (5), takes at the discrete time moments 0, T, 2T, . . . the form

$$\Delta(e^{pT}) = \begin{vmatrix} \dfrac{e^{pT} - e^{\mu_1T_1+\lambda_1T_2}}{\mu_1-\lambda_1} & \dfrac{e^{pT} - e^{\mu_2T_1+\lambda_1T_2}}{\mu_2-\lambda_1} & \cdots & \dfrac{e^{pT} - e^{\mu_mT_1+\lambda_1T_2}}{\mu_m-\lambda_1} \\ \dfrac{e^{pT} - e^{\mu_1T_1+\lambda_2T_2}}{\mu_1-\lambda_2} & \dfrac{e^{pT} - e^{\mu_2T_1+\lambda_2T_2}}{\mu_2-\lambda_2} & \cdots & \dfrac{e^{pT} - e^{\mu_mT_1+\lambda_2T_2}}{\mu_m-\lambda_2} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{e^{pT} - e^{\mu_1T_1+\lambda_mT_2}}{\mu_1-\lambda_m} & \dfrac{e^{pT} - e^{\mu_2T_1+\lambda_mT_2}}{\mu_2-\lambda_m} & \cdots & \dfrac{e^{pT} - e^{\mu_mT_1+\lambda_mT_2}}{\mu_m-\lambda_m} \end{vmatrix} = 0, \tag{7}$$

where $\mu_1, \mu_2, \ldots, \mu_m$ are the roots of the characteristic equation corresponding to the system (1) for the first values of the parameters, or to the system (5) when $f(t) = 0$, that is they are the roots of the equation(2): $\lambda_1, \lambda_2, \ldots, \lambda_m$ are the roots of the characteristic equation corresponding to the system (1) for the other values of the parameters, or to the system (5) when $f(t) = 1$, that is they are the roots of the equation

$$\begin{vmatrix} \lambda - b_{11} - h_1\gamma_1 & -b_{12} - h_1\gamma_2 & \ldots & -b_{1m} - h_1\gamma_m \\ -b_{21} - h_2\gamma_1 & \lambda - b_{22} - h_2\gamma_2 & \ldots & -b_{2m} - h_2\gamma_m \\ \cdots & \cdots & \cdots & \cdots \\ -b_{m1} - h_m\gamma_1 & -b_{m2} - h_m\gamma_2 & \ldots & \lambda - b_{mm} - h_m\gamma_m \end{vmatrix} = 0. \tag{8}$$

The characteristic equation (7) is essentially derived as follows [3]. The initial form (5) of the equation is reduced to A. I. Lur'e canonical form for the time interval $nT < t < nT + T_1$ and independently also for $nT + T_1 < t < nT + T$. A system of difference equations is found between the canonical variables at the discrete instances nT and $(n + 1)T$; these difference equations are obtained by adjusting the solutions of the canonical equations in the above time intervals also taking into account the relations between the canonical variables. The characteristic equation (7) is now obtained in the usual manner from the system of difference equations.

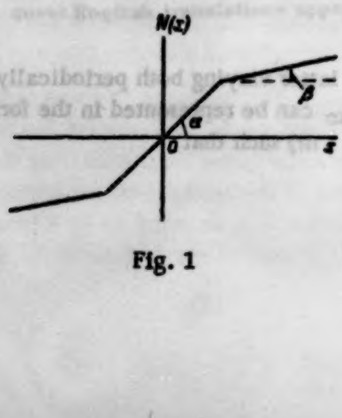

Fig. 1

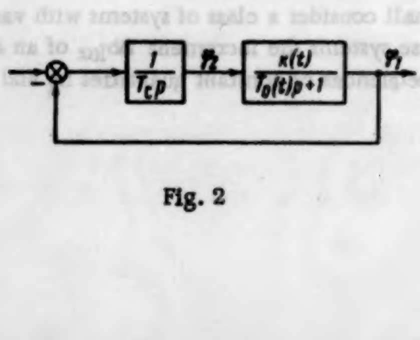

Fig. 2

In order that the system with variable parameters be stable, it is necessary that the real parts of the roots p_1, $p_2, \ldots, p_m$ of the equation (7) be negative.

Thus, if the original system (1) with variable parameters can be brought to the form (5), the following steps should be taken when investigating its stability:

1) to find the roots of the system characteristic equation for one set of the values of the parameters: $\mu_1, \mu_2, \ldots, \mu_m$;

2) to find the roots of the characteristic equation for the other set of the values of the parameters: $\lambda_1, \lambda_2, \ldots, \lambda_m$;

3) to substitute these values into the characteristic equation (7) given in its determinational form;

4) to represent the characteristic equation (7) in the form of a polynomial of the variable e^{pT} by expanding the determinant, and subsequently to apply any of the stability criteria for pulse automatic systems as given in [5].

The restricting condition (4) is satisfied in a wide range of problems of practical interest. This is particularly true for closed-loop systems consisting of a constant parameter part of arbitrary order in cascade with an aperiodic member whose parameters, the gain and the time constant behave in the periodic stepwise pattern.

The results obtained can also be applied in order to study the stability of auto-oscillations in a nonlinear system with a broken line nonlinearity N, as shown in Fig. 1. It is known [6, 7] that the investigation of stability of auto-oscillations in systems of this kind can be reduced to the study of stability of the equilibrium position of a system with periodically varying parameters. In such a case, the new system is obtained by replacing the nonlinearity N by an amplifier whose gain assumes periodically the values $\tan\alpha$ and $\tan\beta$ (with the period equal to the period of auto-oscillations). Such an equivalent system can be described by simultaneous equations which can easily be represented in the form (5).

EXAMPLE

We shall consider a typical automatic control system consisting of one integrator and one aperiodic member (Fig. 2). The equations of the system (in the absence of a disturbance) are

$$T_0(t)\,\varphi_1' + \varphi_1 = k(t)\,\varphi_2, \quad T_c\varphi_2 = -\varphi_1$$

or

$$\varphi_1' = p_1(t)\,\varphi_1 + k_1(t)\,\varphi_2, \quad \varphi_2' = k_2\varphi_1, \tag{9}$$

where

$$p_1(t) = -\frac{1}{T_0(t)}, \quad k_1(t) = \frac{k(t)}{T_0(t)}, \quad k_2 = -\frac{1}{T_c}.$$

Let the time constant $T_0(t)$ and gain $k(t)$ vary periodically and stepwise. We denote the increments of the parameters p_1 and k_1 due to the increments of the parameters T_0 and K by Δp_1 and Δk_1. By putting

$$\gamma_1 = \Delta p_1, \quad \gamma_2 = \Delta k_1, \quad h_1 = 1, \quad h_2 = 0,$$

we obtain the system (9) in the form (5). Consequently in view of (7), the characteristic equation can, in this case, be written in the form

$$\begin{vmatrix} \dfrac{e^{pT} - e^{\mu_1 T_1 + \lambda_1 T_2}}{\mu_1 - \lambda_1} & \dfrac{e^{pT} - e^{\mu_2 T_1 + \lambda_1 T_2}}{\mu_2 - \lambda_1} \\ \dfrac{e^{pT} - e^{\mu_1 T_1 + \lambda_2 T_2}}{\mu_1 - \lambda_2} & \dfrac{e^{pT} - e^{\mu_2 T_1 + \lambda_2 T_2}}{\mu_2 - \lambda_2} \end{vmatrix} = 0, \tag{5}$$

where μ_1 and μ_2 are the roots of the system characteristic equations for one set of values of the parameters and for the other set.

We put T = 1 sec, $T_1 = T_2 = 0.5$ sec, $T_c = 1$ sec.

$$k(t) = \begin{cases} 0.9 & \text{for} \quad n < t < n + 0.5, \\ 1.2 & \text{for} \quad n + 0.5 < t < n + 1, \end{cases}$$

$$T_0(t) = \begin{cases} 0.1 & \text{for} \quad n < t < n + 0.5, \\ 0.2 & \text{for} \quad n + 0.5 < t < n + 1. \end{cases}$$

Then for $n < t < n + 0.5$ the characteristic equation becomes

$$\begin{vmatrix} 0.1\mu + 1 & -0.9 \\ 1 & \mu \end{vmatrix} = 0.1\mu^2 + \mu + 0.9 = 0,$$

that is $\mu_1 = -1$, $\mu_2 = -9$.

For $n + 0.5 < t < n + 1$

$$\begin{vmatrix} 0.2\lambda + 1 & -1{,}2 \\ 1 & \lambda \end{vmatrix} = 0.2\lambda^2 + \lambda + 1.2 = 0,$$

that is $\lambda_1 = -2$, $\lambda_2 = -3$.

By substituting these numerical values into (10) we obtain the following characteristic equation

$$\begin{vmatrix} \dfrac{e^p - e^{-0.5-1}}{-1+2} & \dfrac{e^p - e^{-4.5-1}}{-9+2} \\ \dfrac{e^p - e^{-0.5-1.5}}{-1+3} & \dfrac{e^p - e^{-4.5-1.5}}{-9+3} \end{vmatrix} = 0$$

or

$$4e^{2p} - 1.158\,e^p + 0.00222 = 0.$$

In order that the roots of the equation

$$a_2 e^{2pT} + a_1 e^{pT} + a_0 = 0$$

be negative the following inequalities must be valid [5].

$$a_2 + a_1 + a_0 > 0, \quad a_2 - a_1 + a_0 > 0, \quad a_2 - a_0 > 0.$$

In our case $a_2 = 4$, $a_1 = -1.158$, $a_0 = 0.00222$. This implies that the varying-parameter system of our example is stable.

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DETERMINING THE TRANSFER FUNCTIONS OF MULTI-DIMENSIONAL LINEAR SYSTEMS FROM THE STATISTICAL CHARACTERISTICS OF THE INPUT AND OUTPUT QUANTITIES OF THE SYSTEMS

J. Matyás and J. Silhánek

(Pardubice, Czechoslovakia)

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The method described in this paper makes it possible to find a mathematical description of linear systems from the spectral densities of the random input and output processes. If internal noise sources are present in the system, then the proposed method can be used to determine the spectral density matrix for the noise at the system output. Thus it is possible to determine the effect of internal noise sources on the behavior of the system.

In investigating automatic control systems using simulators it is necessary to know the transfer functions or the differential equations for the system.

In practice it is often required to investigate systems whose mathematical description is unknown. Finding such a mathematical description is in many cases a rather complex problem.

The input and output quantities for such systems can be treated as stationary random functions. The method described in this paper can be used to determine the transfer functions of the investigated dynamic systems from the statistical properties of the input and output quantities.

If the investigated systems are not rigorously (exactly) linear and if their exact behavior cannot be described simply, then the proposed method makes it possible to find the linear approximation for these systems.

Statement of the Problem

Systems with several inputs and outputs shall be called multidimensional systems. Figure 1 shows a multidimensional system with $\underline{n}$ inputs and $\underline{m}$ outputs. Assume that stationary random processes (a random vector) act at the inputs of this system:

$$v = [v_1, v_2, \ldots, v_n].$$

Assume that the system contains internal noise sources which are uncorrelated (for example, statistically independent) with respect to the input processes. Then the output random vector

$$u = [u_1, u_2, \ldots, u_m]$$

can be described by a sum

$$u = x + y = [x_1 + y_1, x_2 + y_2, \ldots, x_m + y_m], \tag{1}$$

where $\underline{x}$ are the output components without the noise and $\underline{y}$ are the components produced by the internal noise sources in the system. Thus the investigated system can be represented in the form of the equivalent block diagram shown in Fig. 2.

We shall use

$$S = [S_{ik}] \qquad (i,\ k = 1,\ 2,\ \ldots,\ n) \tag{2}$$

to denote the spectral density matrix for the input quantities (the input random vector) $\underline{v}$. The expression

$$G = [G_{ik}] \qquad (i,\ k = 1,\ 2,\ \ldots,\ m) \tag{3}$$

shall denote the spectral density matrix for the output quantities (the output random vector) $\underline{u}$, and

$$H = [H_{ik}] \qquad (i = 1,\ 2,\ \ldots,\ n;\ k = 1,\ 2,\ \ldots,\ m) \tag{4}$$

denotes the matrix for the mutual spectral densities of the random vectors $\underline{v}$ and $\underline{u}$ (cf. [1]).

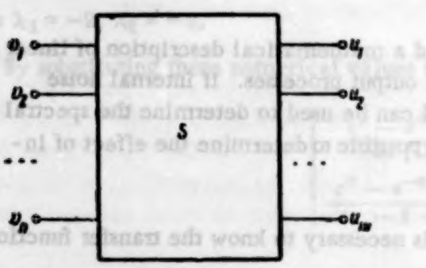

Fig. 1. Multidimensional system of the general type.

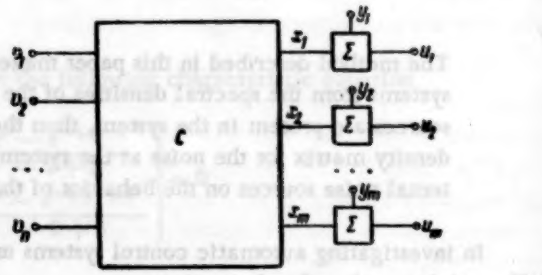

Fig. 2. Equivalent block diagram for a multidimensional system containing internal noise sources.

Fig. 3

Assume that the investigated system is a multidimensional linear system of the general type. A portion of this system (that corresponding to the $\underline{k}$th output) is shown in Fig. 3.

The transfer functions Y_{ik} of the investigated system form the matrix

$$Y = [Y_{ik}] \qquad (i = 1,\ 2,\ \ldots,\ n;\ k = 1,\ 2,\ \ldots,\ m). \tag{5}$$

Our problem now consists of the following.

1. We must determine the matrix Y for the transfer functions of the investigated system C from the specified spectral density matrices S, H, G.

2. We must establish whether or not internal noise sources exist in the system C.

3. In the case $y \neq 0$ we must determine the spectral density matrix for the vector $\underline{y}$ and construct shaping filters for generating the random vector $\underline{y}$ (the random quantities $y_1, y_2, \ldots, y_m$ which are the components of the vector $\underline{y}$).

The Fundamental Matrix Relationship

Since the random vectors $\underline{v}$ and $\underline{y}$ are uncorrelated, it is not difficult to prove that the matrix H (4) is also the matrix for the mutual spectral densities of the vectors $\underline{v}$ and $\underline{x}$. Therefore [cf. [1], formula (48)] the matrices S, H, Y are related by the expression

$$SY = H \tag{6}$$

If the random quantities v_i (the components of the input vector $\underline{v}$) are not mutually correlated, then the matrix S is a diagonal matrix.

Therefore in this case the transfer functions Y_{ik} are determined very simply from (6):

$$Y_{ik} = \frac{H_{ik}}{S_{ii}}. \tag{7}$$

In the general case the matrix relationship (6) represents $\underline{m}$ systems of linear algebraic equations with a general matrix S for the system and $\underline{n} \cdot \underline{m}$ unknowns Y_{ij}. The system (6) has a unique solution only in the case where the determinant for the system is non-zero (i.e., when the matrix S for the system is non-singular). Since S is a Hermitian matrix, it follows that the uniqueness condition for the solution of (6) reduces to the form

$$|S| > 0. \tag{8}$$

In this paper we shall limit ourselves to the case of a non-singular matrix S [i.e., we shall assume that condition (8) is satisfied]. In that case there exists an inverse matrix S^{-1} and the solution of the matrix equation (6) is determined from the formula

$$Y = S^{-1}H \tag{9}$$

For the case of a singular matrix $S(|S| = 0)$ either Eq. (6) is contradictory, or an infinite set of solutions exists. The transfer function Y_{ik} of the investigated system cannot be determined in single-valued fashion in this case. Therefore this case is not of interest as far as practical applications are concerned.

The Solution of the Matric Equation (6)

The matrix equation (6) represents $\underline{m}$ system of linear algebraic equations with an over-all system matrix S with $\underline{n} \cdot \underline{m}$ unknowns. In order to solve it we may use different methods cited in the literature (cf., for example, [3]). Thus by applying Cramer's rule we obtain each transfer function Y_{ik} in the form of a ratio of determinants. But the computation of higher order determinants whose elements are the functions of a complex variable requires difficult computations. The over-all matrix S for the system (6) is by definition a non-singular Hermitian matrix. This fact permits considerable simplification of the process involved in the solution of the problem. It is not difficult to prove that the matrix S can be written in the form of the product

$$S = \overline{B}'B, \tag{10}$$

where B is a triangular matrix. The matrix S can be reduced to the form (10) using, for example, the method described in [6, 7]. The elements B_{ik} of the matrix B are, with the exception of phase filters, the transfer functions of the system of shaping filters which generates the random processes with the spectral density matrix S. The matrix $\overline{B}'$ is the transposed matrix with conjugate elements relative to B.

Therefore the matrix equation (6) can be replaced by the equivalent system of equations:

$$\overline{B}'X = H, \qquad BY = X. \tag{11}$$

Since the matrix B, and therefore $\overline{B}'$, are triangular matrices, it follows that the solution of matrix equations (11) is not difficult. First of all we use the first equation to compute the auxiliary matrix S, and then use the second equation to compute the desired matrix Y for the transfer functions of the investigated system C. In view of (10) it is possible to write the inverse matrix S^{-1} in the form

$$S^{-1} = B^{-1}(\overline{B}')^{-1}. \tag{12}$$

Thus by substitution into (9) we obtain the solution of the fundamental matrix relationship (6) in the form

$$Y = B^{-1}(\overline{B}')^{-1}H. \tag{13}$$

Formula (13) can be used expediently for solving the matrix equation (6), since Eq. (6) represents m systems of linear algebraic equations with an over-all matrix S, and the inversion of the triangular matrices B and B' is not difficult.

At the beginning of this paper the problem was limited to an investigation of linear stable systems whose input and output quantities are stationary random processes. We also adopted condition (8) which requires that the matrix S must be non-singular.

Assuming that these conditions are satisfied, it is possible to use the method described above to determine the matrix Y for the transfer functions of the investigated system. This matrix describes the behavior of the investigated system. This matrix describes the behavior of the investigated system exactly. Such a statement is valid only on the assumption that all of the measurements and computations which are used to determine the spectral densities S_{ik} and H_{ik} are performed exactly, and that the computations used to solve the system (6) are also formed exactly. In practice it is necessary first of all to use a special computer (correlator) to find all the required correlation functions; then some approximate Fourier transform method is used to compute the corresponding spectral densities. These measurements and computations cannot be performed exactly in practice. Thus, the limitations cited above will not be rigorously satisfied. Therefore the solution Y in this case determines a certain linear approximation of the mathematical description of the investigated system.

Internal Noise Sources

If the system C contains internal noise sources, then it is possible to determine the matrix for the spectral densities of the random vector y whose components y_i correspond to the noise signals at the system output.

By definition the vectors v and y are uncorrelated; i.e., the matrix for the mutual correlation functions will be a zero matrix (cf. [1]):

$$M\{v(t)'\,y(t+\tau)\} \equiv 0 \tag{14}$$

(here M denotes mathematical expectation). It is not difficult to demonstrate the fact that the matrix

$$M\{x(t)'\,y(t+\tau)\} \equiv 0, \tag{15}$$

will also be a zero matrix; i.e., the random vectors x and y are also uncorrelated.

In this case the matrix G can be written in the form

$$G = G_x + G_y, \tag{16}$$

where G_x and G_y are the matrices for the spectral densities of the random vectors x and y.

The matrix G_x can be written as (cf. [1])

$$G_x = \overline{Y}'SY. \tag{17}$$

The matrices S, G, Y are assumed known, and therefore we determine the matrix G_y from the formula

$$G_y = G - G_x = G - \overline{Y}'SY. \tag{18}$$

Formula (18) determines the spectral density matrix for the random vector y corresponding to the noise signals at the system output. The shaping filters which generate the quantities y_i with the specified spectral density matrix G_y can be determined using the methods described in [6, 7].

Example

It is required to determine the matrix Y for the transfer functions of a linear system with two inputs and three outputs from the specified spectral density matrices:

$$S = \begin{bmatrix} \dfrac{1}{4-s^2} & \dfrac{1}{(1-s)(2-s)} \\ \dfrac{1}{(1+s)(2+s)} & \dfrac{2}{1-s^2} \end{bmatrix},$$

$$H=\begin{bmatrix}\frac{1}{(1+s)(2-s)} & \frac{1}{4-s^2} & \frac{2}{(3+s)(2-s)}\\ \frac{1}{(1+s)(2+s)} & \frac{1}{(1+s)^2} & \frac{1}{(3+s)(1+s)}\end{bmatrix},\quad s=j\omega.$$

First we determine the matrix B by the method described in [6]:

$$B=\begin{bmatrix}\frac{1}{2+s} & \frac{1}{1-s}\\ 0 & \frac{1}{1+s}\end{bmatrix}$$

The transfer functions Y_{lk} could be computed by solving the system of matrix equations (11). Here it is expedient to use formula (13) to solve the problem.

We shall compute the reciprocal matrix

$$B^{-1}=\begin{bmatrix}2+s & -\frac{1+s}{1-s}(2+s)\\ 0 & 1+s\end{bmatrix}$$

and from formula (12) we obtain

$$S^{-1}=\begin{bmatrix}2(4-s^2) & -(1+s)(2+s)\\ -(1-s)(2-s) & 1-s^2\end{bmatrix}.$$

Substituting into (9), we find

$$\begin{bmatrix}\frac{3+s}{1+s} & \frac{s}{1+s} & \frac{3(2+s)}{3+s}\\ -\frac{1-s}{(1+s)(2+s)} & \frac{1-s}{(1+s)(2+s)} & -\frac{1-s}{3+s}\end{bmatrix},$$

and thus we have determined the mathematical description of the investigated linear system.

In the case where the spectral density matrix G for the output random vector u is also specified, we first of all establish whether or not internal noise sources exist in the system. If they exist, then from formula (18) we determine the spectral density matrix G_y for the random vectory y (the output noise signals of the system). The shaping filters which generate the corresponding random processes can be defined and constructed by the methods described in [6, 7].

SUMMARY

The paper describes the method for determining the matrix for the transfer functions of multidimensional linear systems from the spectral densities of the random input and output quantities. These transfer functions are the solution of the matrix equation

$$SY = H$$

i.e., they are a solution of m systems of linear algebraic equations with an overall system matrix S.

If the system contains internal noise sources, then the method described above can be used to compute the spectral density matrix for the random vector y (the output components of this vector correspond to the internal noise sources in the investigated system).

In practice this method makes it possible to compute the linear approximation of multidimensional systems whose mathematical description is unknown.

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EFFECT OF COULOMB FRICTION IN GUIDES ON THE STABILITY OF DUPLICATING-MACHINE HYDRAULIC SERVOSYSTEMS

B. L. Korobochkin and A. I. Levin

(Moscow)

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The present article is concerned with the effect of Coulomb friction and of the feed pump capacity limitations on the stability of duplicating-machine hydraulic servosystems. The solution which was obtained by means of a mock-up of the device showed that the previously used solution, which was given by A. A. Andronov and A. G. Maier, is incorrect in this case because of the difference between the initial conditions. The solution obtained here makes it possible to render the known methods for calculating hydraulic servosystems more accurate.

Coulomb friction in the guides of operating parts exerts a considerable influence of the stability of hydraulic duplicating machines. An attempt was made in [1] to take into account the Coulomb friction forces in deriving the stability conditions for duplicating machines. In this, the results obtained by A. A. Andronov and A. G. Maier [2] in applying the direct control theory were used.

E. M. Khaimovich [3] and T. M. Bashta [4] later used the same results. However, further investigations showed that, strictly speaking, the direct application of the results obtained by A. A. Andronov and A. G. Maier to the problem of the stability of a hydraulic servosystem was not justified due to the difference between the initial conditions, although the obtained stability condition was in many cases in good agreement with the results obtained in practice.

In connection with this, it became necessary to analyze this problem more rigorously and to formulate the stability conditions in a more precise manner. For a more accurate description of the system's behavior under oscillating conditions, the characteristic of hydraulic drive saturation, which is determined by the finite capacity of the pump feeding the system, was introduced in the consideration. An MN-M electronic simulating device was used in solving this problem.

For checking the accuracy of the obtained solution, we used the same mock-up for producing the solution of Vyshnegradskii's problem, which coincided with the exact solution given by A. A. Andronov and A. G. Maier.

The derivation of the basic equation of motion is given in [1]. Here, we shall consider only the effect of Coulomb friction and of the saturation characteristic, which is determined by the pump capacity limitations.

The linearized equation of the system for a linear input function, i.e., for a constant velocity of the pickup end-piece, where the friction force is not taken into account [1], has the following form:

$$L\frac{d^3\delta}{dt^3} + M\frac{d^2\delta}{dt^2} + D\frac{d\delta}{dt} + E\delta = D\frac{dy}{dt} + R. \tag{1}$$

Here, L is a coefficient which characterizes the effect of the compressibility of the operating liquid and of the elasticity of piping, M is the mass of the machine's operating element, D is the internal damping coefficient, E is the rigidity coefficient, R is the constant component of the cutting force, δ is the error in duplicating, and y is the input coordinate (the coordinate of the pickup end-piece);

$$\delta = y - x,$$

where x is the coordinate of the machine's operating element.

The problem of the stability of this system can be reduced to Vyshnegradskii's problem in the theory of direct control. The Vyshnegradskii parameters for the system under investigation are

$$z = \frac{D}{\sqrt[3]{E^2L}}, \quad U = \frac{M}{\sqrt[3]{EL^2}}. \tag{2}$$

The stability condition is given by the inequality $UZ \geq 1$, which corresponds to the points that lie in the plane of parameters U and Z above the Vyshnegradskii hyperbola and on it.

Let us consider the system's behavior in the presence of Coulomb friction in the case where it moves about the equilibrium position.

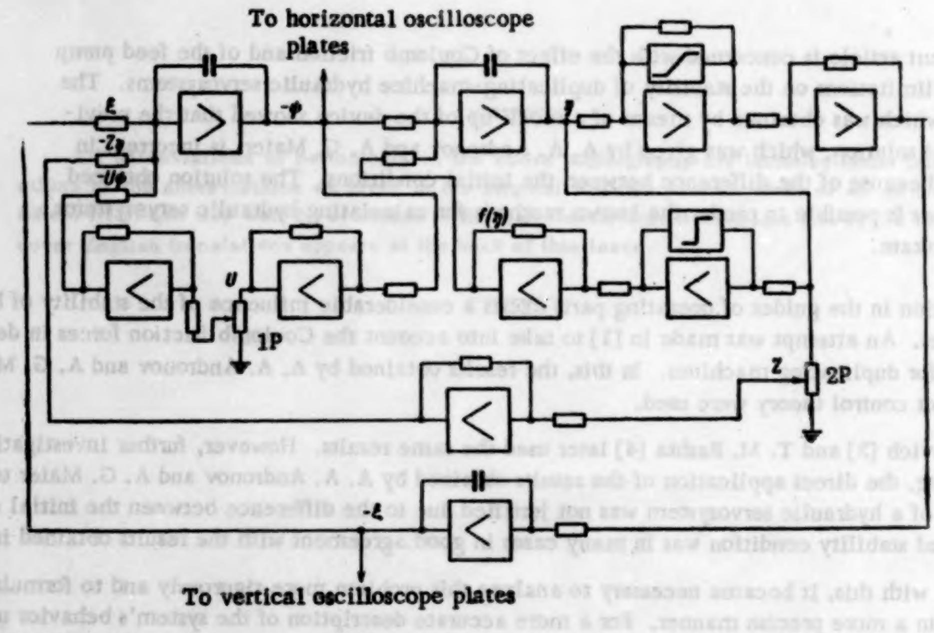

Fig. 1. Simulation scheme.

Assuming that R = 0 and y = 0, the system's behavior can be described by the following system of equations:

$$M\frac{d^2x}{dt^2} = P + f(x'), \quad \frac{L}{M}\frac{dP}{dt} + P = -Ex - D\frac{dx}{dt}, \tag{3}$$

where P is the motive force;

$$f(x') = \begin{cases} +\frac{P_T}{2} & \text{for } x' < 0 \text{ or} \\ & \text{for } x' = 0 \text{ and } |P| < \frac{P_T}{2}, \\ -\frac{P_T}{2} & \text{for } x' > 0 \text{ or} \\ & \text{for } x' = 0 \text{ and } |P| > \frac{P_T}{2}, \\ -P & \text{for } x' = 0 \text{ and } |P| < \frac{P_T}{2} \end{cases} \tag{4}$$

For the sake of brevity, we shall denote this function by the symbol

$$f(x') = -\frac{P_\tau}{2}\,\mathrm{sign}\,x'.$$

By dividing the second equation in system (3) by E, we obtain:

$$\frac{L}{ME}\frac{dP}{dt} + \frac{1}{E}P = -x - \frac{D}{E}\frac{dx}{dt}.$$

We shall now reduce the system under investigation to the dimensionless form by substituting the variables according to the method used by Vyshnegradskii and by A. A. Andronov and A. G. Maier:

$$x = \gamma\xi,\quad P = \gamma M\sqrt[3]{\frac{E_2}{L_2}}\,\psi,\quad P_\tau = \gamma M\sqrt[3]{\frac{E_2}{L_2}},\quad t = \sqrt[3]{\frac{L}{E}}\,\tau. \tag{5}$$

Here ξ and ψ are the dimensionless coordinates of the system, and τ is the dimensionless time.

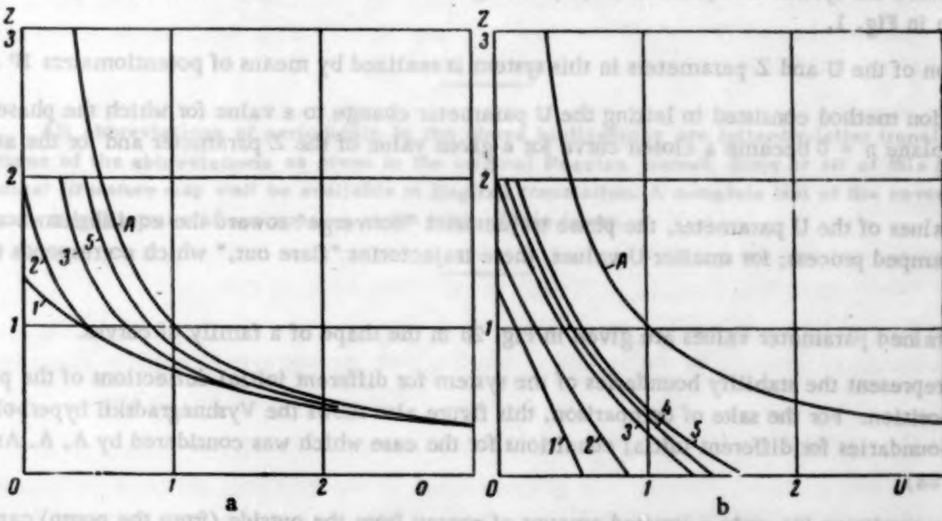

Fig. 2. a) Boundaries of the stability region for different initial conditions for the case considered by A. A. Andronov and A. G. Maier; b) boundaries of the stability region of the duplicating system for different initial deflections of the pickup valve. 1) $\xi_0 = 0.5$; 2) $\xi_0 = 1.0$; 3) $\xi_0 = 2.0$; 4) $\xi_0 = 2.5$; 5) $\xi_0 = 5.0$; A) Vyshnegradskii hyperbola.

After performing the substitution and the necessary simplifications, system (3) assumes the following form:

$$\frac{d^2\xi}{d\tau^2} = \psi - \frac{1}{2}\,\mathrm{sign}\,\frac{d\xi}{d\tau},\quad \frac{d\psi}{d\tau} + U\psi = -\xi - z\frac{d\xi}{d\tau}, \tag{3'}$$

where U and Z are Vyshnegradskii's parameters.

Assuming that $\frac{d\xi}{d\tau}$, we obtain:

$$\frac{d\xi}{d\tau} = \eta,\quad \frac{d\eta}{d\tau} = \psi - \frac{1}{2}\,\mathrm{sign}\,\eta,$$
$$\frac{d\psi}{d\tau} = -\xi - U\psi - Z\eta. \tag{3''}$$

The initial conditions are: $\xi_0 \neq 0$, $\eta_0 = 0$.

Due to the fact that the capacity of the pump that feeds the system is a finite quantity, the velocity of the copying support cannot exceed the value

$$x'_{max} = \frac{Q_p}{F}, \tag{6}$$

where Q_p is the pump capacity and F is the cross-sectional area of the piston, if the condition of the liquid flow continuity is to be preserved.

In dimensionless form,

$$\left(\frac{d\xi}{d\tau}\right)_{max} = \eta_{max} = \frac{Q_p \sqrt[3]{\frac{L}{E}}}{F\gamma}. \tag{7}$$

Thus, the following additional condition is imposed on the solution of system (3"):

$$|\eta| \leqslant \eta_{max}. \tag{8}$$

In order to solve the system of equations (3") by taking into account condition (8), we composed the simulation scheme shown in Fig. 1.

The variation of the U and Z parameters in this system is realized by means of potentiometers 1P and 2P.

The simulation method consisted in letting the U parameter change to a value for which the phase trajectory projection on the plane $\eta = 0$ became a closed curve for a given value of the Z parameter and for the assigned initial conditions.

For larger values of the U parameter, the phase trajectories "converge" toward the equilibrium zone, which corresponds to a damped process; for smaller U values, these trajectories "flare out," which corresponds to a divergent process.

The thus obtained parameter values are given in Fig. 2b in the shape of a family of curves.

Curves 1-5 represent the stability boundaries of the system for different initial deflections of the pickup valve from the neutral position. For the sake of comparison, this figure also shows the Vyshnegradskii hyperbola (Fig. 2b) and the stability boundaries for different initial conditions for the case which was considered by A. A. Andronov and A. G. Maier (Fig. 2a).

According to condition (8), only a limited amount of energy from the outside (from the pump) can be supplied to the system. The energy supplied to the system is regulated by the magnitudes of the cross-sectional areas of the slide valve passages. Beginning with a certain magnitude of the passage cross section, which is determined by the maximum displacement $|\varepsilon_{max}|$, the supply of energy does not change, and, consequently, if the system is stable for $|\varepsilon_{max}|$, it will also be stable for $|\varepsilon| > |\varepsilon_{max}|$. Such a system can be considered as absolutely stable. The value of $|\varepsilon_{max}|$ is given by

$$|\xi_{max}| = Z\eta_{max} + U\psi_{max} \tag{9}$$

By expressing the relative parameters in terms of the absolute parameters, we obtain:

$$|\xi_{max}| = \frac{M}{P_\tau \sqrt[3]{L^2E}} \left(\frac{EQ_p}{FC_0} + P_{max}\right), \tag{10}$$

where, according to the linearization conditions for the discharge equations [1],

$$P_{max} \leqslant 0{,}5\,pF,$$

where p is the pump pressure.

As was shown by mock-up tests, the amplitude of self-oscillations which can arise in the system if it is outside the stability boundaries is directly dependent on the pump capacity.

SUMMARY

The results obtained by A. A. Andronov and A. G. Maier cannot be directly extended to the problem of the stability of hydraulic servosystems in duplicating machines by taking into account Coulomb friction in the guides.

As a result of investigations of a hydraulic servosystem where the limitations with respect to the pump capacity were taken into account, we have derived the absolute stability conditions for the system in dependence on its parameters and the pump capacity, which made it possible to formulate the previously derived stability conditions with a greater degree of accuracy.

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EFFECT OF LEAKAGES ON THE CHARACTERISTICS OF PNEUMATIC FORCE-COMPENSATING UNITS

A. I. Gitel'man and V. M. Syrodoev

(Leningrad)

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This article presents theoretical and experimental data on the operation of a membrane-nozzle-valve complex under air pressures of up to 10 atm. Recommendations concerning the stability of the characteristics are given.

Along with pneumatic automation devices which operate under air pressures of up to 1 atm abs (AUS devices, etc.), devices using pressures of up to 10 atm are also used to a large extent. As a rule, the use of high air pressure is determined by the characteristics of the systems to be automated.

The force-compensating unit is one of the basic units in modern pneumatic automation devices. The complex consisting of a membrane, a nozzle, and a valve represents a widely used basic design of this unit. The operation of this complex at high air pressures is connected with a number of peculiarities which make it different from operation at low pressures.

The present article provides the generalization of some experience gained in factory adjustment of automatic control systems with force-compensating units that operate under a pressure of 10 atm.

Figure 1 shows the schematic diagram of the force-compensating unit. Its operating principle is based on the compensation of the command force Q_c by the force Q_m, which the air pressure p in chamber A exerts on the membrane.*

Let us consider the equilibrium of the forces acting in the force-compensating unit.

The possible force distributions can be reduced to three cases. In the first case, the nozzle is pressed against the valve, and the valve is not in forced contact with the valve seat. In this case, the equilibrium of forces is determined by the relationship

$$Q_c - Q_v - Q_m = 0, \tag{1}$$

where Q_c is the command force, Q_v is the force which acts on the valve from the side of compressed air and which is composed of the spring-action force and the force resulting from the difference between the air pressures acting on the valve, and Q_m is the force which the air pressure p exerts on the membrane.

In the second possible case of force distribution, the valve is pressed against the seat, and the nozzle is not in forced contact with the valve. In this case, the force Q_v does not figure in the balance of forces, and the equilibrium condition is determined by the relationship

$$Q_c - Q_m = 0. \tag{2}$$

In the third possible case of force distribution, the action of force Q_v is divided between pressing the valve against the seat and pressing it against the nozzle.

* The case where the air is not used by a device is considered.

The equilibrium condition is determined by the relationship

$$Q_c - Q_m - (Q_v - Q_s) = 0. \tag{3}$$

Here, Q_s is the force with which the valve is pressed against the sealing edge of the seat.

Let us consider the conditions under which the above cases of force distribution can arise.

For an ideal airtightness of the contact between the valve and the seat and between the valve and the nozzle,* the force distribution that is connected with an increase in the force Q_c within the range $0 < Q_c < Q_{v0}$ (section OC in Fig. 2) corresponds to the third case of force distribution: Q_c is spent on overcoming the force Q_{v0} with which the valve is pressed against the seat.

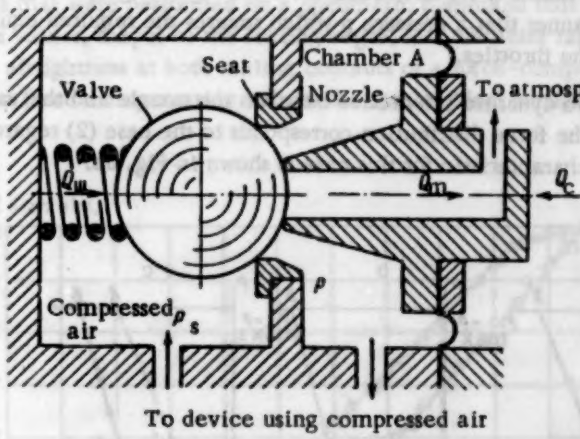

Fig. 1. Schematic diagram of the force-compensating unit.

Due to the ideal airtightness of the valve–seat contact, the air under pressure is prevented from entering chamber A along the section OC, and the air pressure p is equal to zero.

If the force Q_c further increases (section CB), we obtain the first case of force distribution: Under equilibrium conditions the nozzle is pressed against the valve with the force Q_v,** while the force which presses the valve against the seat is $Q_c = 0$. In this case, the increase in pressure p occurs as a consequence of feeding compressed air into chamber A through the clearance between the valve and the seat, which is formed at the instant of time when the force Q_c increases since the relationship $Q_c > Q_m + Q_v$ holds until equilibrium is established.

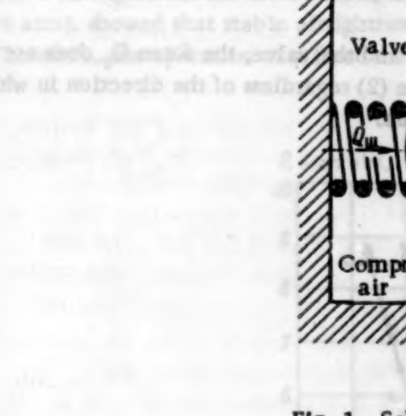

Fig. 2. Static characteristics of the force-compensating unit in the case of ideal airtightness of the sealing edges.

At the point B, pressure p attains the value $p_{max} = p_s$, and the subsequent increase in Q_c (the section to the right from the point B) is spent on compressing the spring. Over this section, the distribution of forces which corresponds to the first case is preserved not only when the force Q_c increases, but also when it decreases.

Along the section BA, we again have the third case of force distribution: A decrease in Q_c is compensated by an increase in the force Q_s with which the valve is pressed against the seat; this force increases to its maximum value $Q_{smax} = Q_{v1}$.

* It is considered that the contact remains airtight even if the force that keeps the sealing edges in contact is equal to zero.

** Since the force Q_v depends on the difference between the air pressures acting on the valve, the pressure difference decreases as the pressure in chamber A increases, while Q_v also decreases $Q_v(Q_{v0} > Q_{v1})$.

In this due to the ideal airthightness of the contact between the valve and the seat and between the valve and the nozzle, the air cannot either enter or leave chamber A, and the pressure is $p = p_{max}$.

If the force Q_c further decreases (section AO), we have the second case of force distribution: Under equilibrium conditions, the valve is pressed against the seat with the force Q_V, while the force with which the nozzle is pressed against the valve is equal to zero. In this case, the pressure is lowered due to the leakage of air from chamber A into the atmosphere through the clearance between the valve and the nozzle, which appears when the force Q_c is reduced, since the relationship $Q_m > Q_c$ applies until equilibrium is established.

Thus, in the case of perfect airtightness, the force-compensating unit is insensitive when force Q_c changes its direction of action. The magnitude of this insensitivity is characterized by the horizontal plateau between the lines OA and CB (for instance, A'C'), i.e., it is equal to the force Q_V.

If the contact between the valve and the seat is not airtight, the valve is pressed against the seat, however, compressed air continuously enters chamber A, and, when the force Q_c changes, the clearance between the nozzle and the valve changes in such a manner that a pressure p which secures the required equilibrium of forces is established in the chamber A between the throttles.

Since, in this case, there is no dynamic interaction between the nozzle and the valve, the force Q_v does not affect the balance of forces, i.e., the force distribution corresponds to the case (2) regardless of the direction in which the force Q_c changes. The static characteristic for this case is shown in Fig. 3a.

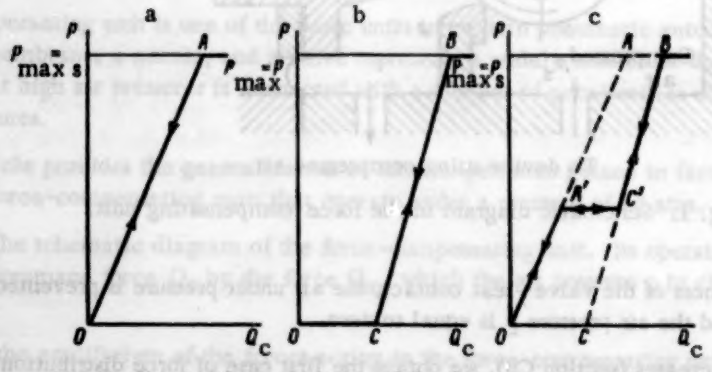

Fig. 3. Static characteristics of the force-compensating unit in the absence of airtightness. a) Leakage through the clearance between the valve and the seat; b) leakage through the clearance between the valve and the nozzle; c) the leakages are commensurable.

In the absence of an airtight contact between the valve and the nozzle, the nozzle is pressed against the valve, however, the air from chamber A continuously escapes into the atmosphere, and, when the force Q_c changes, the necessary equilibrium or forces is secured by varying the clearance between the valve and the seat.

In this case, after the valve moves away from the seat, the nozzle and the valve are continuously in contact due to the pressing force Q_V, i.e., the distribution of forces for $Q_c > Q_V$ (section CB in Fig. 3b) corresponds to the case (1) regardless of the direction in which Q_c changes.

Besides the cases considered above, we can also have the case where both the seat and the nozzle contacts are not airtight. The shape of the force-compensating unit characteristic will be then determined by the ratio of clearance areas which are formed in forced contact between the valve, the seat, and the nozzle. If the clearance area between the valve and the seat is much larger than the clearance area between the valve and the nozzle, the characteristic will correspond to that shown in Fig. 3a. If the clearance area between the valve and the nozzle is much larger, the characteristic will correspond to that given in Fig. 3b. If these clearances are commensurable, the characteristic will consist of the two sections OA' and C'B (Fig. 3c). On the section OA', an increase in the force Q_c leads to a reduction of the clearance between the nozzle and the valve, due to which the pressure p increases. The second case of force distribution applies here. At the point A', the nozzle is brought into forced contact with the valve,

and a further increase in the force Q_c does not affect the ratio of the cross-sectional areas of passages at the inlet and outlet of chamber A. The force Q_c is spent on overcoming the force Q_v. The relationship between forces corresponds to the third case (3).

Along the section C'B, an increase in Q_c leads to an increase in the clearance between the valve and the seat, due to which pressure p further increases. The relationship between forces is determined by expression (1).

Thus, if the clearances between the elements in a force-compensating unit are mutually commensurable, the characteristic can have an insensitivity region, as in the case of ideal airtightness. The magnitude of this insensitivity, which is equal to Q_v, is present in both cases, however, its character is different: If the airtightness is ideal, the insensitivity arises when the direction of force Q_c is reversed, while, when the clearances are commensurable, the insensitivity is determined by their ratio and is independent of the direction in which Q_c changes.

Numerous experiments that were performed on a pneumatic command unit with metallic nozzles and seats, which had different hardness values, shapes of the sealing edges, and diameter ratios (for p_s and p pressures of up to 10 atm), showed that stable airtightness at both sealing contacts of a force-compensating unit cannot be guaranteed. Therefore, the operation of a force-compensating unit in the case of ideal airtightness is mainly of theoretical interest.

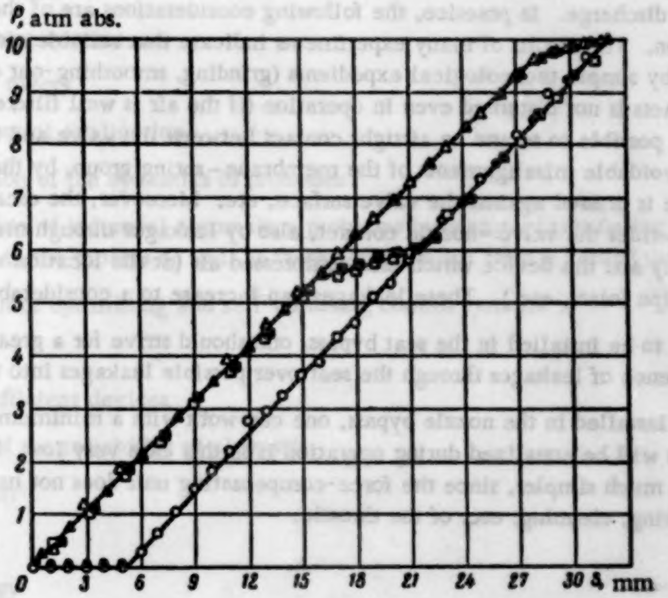

Fig. 4. Static characteristics of an experimental pneumatic command device for different leakage ratios (p_s = 10 atm). The circles correspond to air discharge G_{sea} = 0 through the contact between the valve and the seat and to discharge G_{noz} = 60 liters/min through the valve–nozzle contact; squares: G_{sea} = 16 liters/min and $G_{noz} \approx 20$ liters/min; triangles: G_{sea} = 50 liters/min and $G_{noz} \approx 6$ liters/min. G_{sea} values were measured for p = 0, and G_{noz} values were for p = 10 atm; the white points correspond to an increase in the command spring tension, and the black points correspond to a decrease in the spring tension.

As a result of numerous experiments, it was established that not only the shape, but also the stability of the characteristics, depends on the ratio of leakages in force-compensating units. This applies especially in those cases where the characteristic has an insensitivity section that is located in the operating range, i.e., in the $0 < p < p_s$ range. The values of p and Q_c that correspond to the appearance of insensitivity and also the slope of the insen-

sitivity section vary in a random fashion for different measurements without any external causes. This can be explained by changes in the conditions which determine the ratio of leakages (clogging and division of the clearance, nonalignment, etc.).

Figure 4 shows the static characteristics $p = f(S)$ of an experimental pneumatic device for different ratios of leakages through the sealing contacts in the force-compensating unit. (Here, S is the deflection of the compressed command spring. The command force is $Q_c = cS$, where $\underline{c}$ is the spring stiffness.) In our experiments, $\underline{c}$ was equal to 1.76 kg/mm, and the compressed-air pressure was $p_s = 10$ atm. The leakages into the seat and nozzle bypass were secured by suitably mounting special adjusting screws.

If air leakages at the seat or at the nozzle are positively provided, it is possible to secure a stable linear characteristic of the force-compensating unit throughout the entire $0 < p < 10$ atm range without any appreciable insensitivity.

From what has been said above, it follows that, for a reliable operation of the force-compensating unit, it is desirable to provide a throttle in the seat or the nozzle bypass, whereby the existence of leakages would be guaranteed.

In choosing the throttle location (in the nozzle or the seat bypass), the requirements for the force-compensating unit characteristic must be taken into account. Moreover, the ratio of the pressure p_s and $\underline{p}$ for the longest operating periods must be considered, since this will determine the throttle dimensions and design in dependence on the maximum allowable air discharge. In practice, the following considerations are of the greatest importance in selecting the throttle location. The results of many experiments indicate that reliable airtightness of the valve-seat contact can be secured by simple technological expedients (grinding, smoothing-out of sharp edges, etc.). In this, the airtightness of contacts is not disturbed even in operation (if the air is well filtered and the valve does not vibrate). As a rule, it is not possible to secure an airtight contact between the valve and the nozzle. This can obviously be explained by unavoidable misalignments of the membrane–spring group, by the nonuniform pressure with which the nozzle's edge is pressed against the valve surface, etc. Moreover, the escaping of the air into the atmosphere is determined, besides the valve–nozzle contact, also by leakages through many seals of the air duct between the membrane cavity and the device which uses compressed air (at the location where the membrane is built in, at the connecting pipe joints, etc.). These leakages can increase to a considerable extent during operation.

Thus, if the throttle is to be installed in the seat bypass, one should strive for a greater air discharge in order reliably to secure the prevalence of leakages through the seat over possible leakages into the atmosphere.

If the throttle is to be installed in the nozzle bypass, one can work with a minimum air discharge, since the probability that the leakages will be equalized during operation is in this case very low. Moreover, the servicing of the throttle then becomes much simpler, since the force-compensating unit does not have to be disconnected and dismantled for the checking, cleaning, etc. of the throttle.

INFORMATION

Translated from Avtomatika i Telemekhanika, Vol. 22, No. 9,
p. 1262, September, 1961

The regular session of the IFAC Executive Council under the Chairmanship of the IFAC President, Prof. A. M. Letov, was held in March 1961 in Bergen. The Executive Council adopted the following resolution concerning the organization of the Second IFAC Congress.

The congress will be held in Basel (Switzerland) in September, 1963.

A. The program of the Second Congress is concerned with the following scientific fields.

1. Automatic control theory:

a) discrete systems

b) stochastic processes

c) optimal systems

d) self-adjusting systems

e) reliability theory

2. Automatic control application:

a) investigation of the dynamics of processes

b) investigation of industrial automation problems by means of simulating devices and digital computers which are included in the process as well as those which do not form a part of the process

c) application of optimizing and self-adjusting control systems

3. Elements:

a) new and efficient devices

b) estimate of the reliability of elements

4. Other subjects:

a) education

b) terminology

c) bibliography.

This program does not exclude other possible subjects; however, not less than 80% of all reports must pertain to the theory and application, and not more than 20% must be devoted to elements and other subjects.

B. The total number of reports submitted to the Congress must not exceed 100. The volume of each report must not exceed 30,000 type-written letters, including a summary in two or three languages and illustrations.

C. The selection of reports will be performed by the IFAC National Committees, which will be entrusted with:

a) contacting various specialists and requesting them to send in reports on the above subjects,

b) finding specialists who will be able to make contributions and to perform the preliminary selection of reports,

c) to inform the authors that the reports must be submitted to the National Committee not later than the end of 1961, and

d) to inform the authors that the reports may be written in Russian, English, French, or German; every report must include a summary of not more than 200 words in the language used in the report as well as in Russian or English.

The final selection of the reports which will be presented to the Congress will be made by the IFAC Committee on Reports, which will be formed by the presidents of the IFAC technical committees under the chairmanship of Prof. Goerecke. In this, the IFAC Committee on Reports will be guided by the following principles in making the decisions:

1) the timeliness of the topic,

2) the value of the obtained results and their novelty, and

3) clarity of presentation.

In connection with the above resolution of the IFAC Executive Council, the National Committee of the USSR on Automatic Control hereby asks the Soviet specialists to send their suggestions to the National Committee (15a Kalanchevskaya Street, I-53, Moscow).

National Committee of the USSR for Automatic Control.